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CBSE Examination Paper, 2020

Time Allowed: 3 Hours] [Maximum Marks: 80

General Instructions:

(i) This question paper comprises four sections – A, B, C and D. This question paper carries 40 questions. All questions are compulsory.

(ii) Section A : Q. No. **1** *to* **20** *comprises of* **20** *questions of one mark each.*

(iii) Section B : Q. No. **21** *to* **26** *comprises of* **6** *questions of two marks each.*

(iv) Section C : Q. No. **27** *to* **34** *comprises of* **8** *questions of three marks each.*

(v) Section D : Q. No. **35** *to* **40** *comprises of* **6** *questions of four marks each.*

(vi) There is no overall choice in the question paper. However, an internal choice has been provided in **2** *questions of one mark each,* **2** *questions of two marks each,* **3** *questions of three marks each and* **3** *questions of four marks each. You have to attempt only one of the choices in such questions.*

(vii) In addition to this, separate instructions are given with each section and question, wherever necessary.

(v) Use of calculators is not permitted.

SET-1

SECTION – A

Question numbers **1** *to* **10** *are multiple choice type questions of* **1** *mark each. Select the correct option.*

1. If one of the zeroes of the quadratic polynomial $x^2 + 3x + k$ is 2, then the value of k is

(*a*) 10 **(*b*) –10** **(*c*) –7** **(*d*) –2**

Sol. (*b*), as one zero is 2, $\Rightarrow (2)^2 + 3 \times 2 + k = 0$

$\Rightarrow k = -4 - 6 = -10$

Alternatively Let other zero be α.

$\therefore \alpha + 2 = -3$ and $2\alpha = k$

$\Rightarrow \alpha = -5$ and $2 \times (-5) = k \Rightarrow k = -10$

2. The total number of factors of a prime number is

(*a*) 1 **(*b*) 0** **(*c*) 2** **(*d*) 3**

Sol. (*c*), total number of factors of a prime number is 2.

3. The quadratic polynomial, the sum of whose zeroes is –5 and their product is 6, is

(*a*) $x^2 + 5x + 6$ **(*b*) $x^2 - 5x + 6$** **(*c*) $x^2 - 5x - 6$** **(*d*) $-x^2 + 5x + 6$**

Sol. (*a*), sum of zeroes = –5, product = 6

Polynomial is, x^2 – (sum of zeroes) x + product of zeroes

$\Rightarrow x^2 - (-5)x + 6 = x^2 + 5x + 6.$

4. The value of k for which the system of equations $x + y - 4 = 0$ and $2x + ky = 3$, has no solution, is

(*a*) –2 **(*b*) $\neq 2$** **(*c*) 3** **(*d*) 2**

Sol. (*d*), For no solution $\frac{1}{2} = \frac{1}{k} \neq \frac{-4}{-3}$

$\Rightarrow k = 2.$

5. The HCF and the LCM of 12, 21, 15 respectively are

(*a*) 3, 140 **(*b*) 12, 420** **(*c*) 3, 420** **(*d*) 420, 3**

Sol. (*c*), as

$$12 = 2^2 \times 3$$
$$21 = 3 \times 7$$
$$15 = 3 \times 5$$
$$\text{HCF} = 3$$
$$\text{LCM} = 2^2 \times 3 \times 7 \times 5 = 420$$

6. The value of x for which $2x$, $(x + 10)$ and $(3x + 2)$ are the three consecutive terms of an AP, is

(*a*) 6 **(*b*) –6** **(*c*) 18** **(*d*) –18**

Sol. (*a*), if $2x$, $(x + 10)$ and $(3x + 2)$ are three consecutive terms of an AP

then $2(x + 10) = 2x + 3x + 2$

$\Rightarrow$ $2x + 20 = 5x + 2 \Rightarrow 3x = 18 \Rightarrow x = 6$

7. The first term of an AP is p and the common difference is q, then its 10th term is

(*a*) $q + 9p$ **(*b*) $p - 9q$** **(*c*) $p + 9q$** **(*d*) $2p + 9q$**

Sol. (*c*), as $a_{10} = p + (10 - 1)q = p + 9q$

8. The distance between the points ($a\cos\theta + b\sin\theta$, 0) and (0, $a\sin\theta - b\cos\theta$), is

(*a*) $a^2 + b^2$ (*b*) $a^2 - b^2$ (*c*) $\sqrt{a^2 + b^2}$ (*d*) $\sqrt{a^2 - b^2}$

Sol. (*c*), as distance $= \sqrt{(a\cos\theta + b\sin\theta - 0)^2 + (0 - a\sin\theta + b\cos\theta)^2}$

$= \sqrt{a^2\cos^2\theta + b^2\sin^2\theta + 2ab\sin\theta\cos\theta + a^2\sin^2\theta + b^2\cos^2\theta - 2ab\sin\theta\cos\theta}$

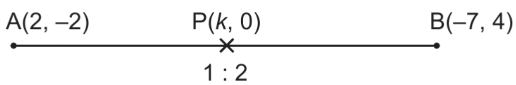
$= \sqrt{a^2(\cos^2\theta + \sin^2\theta) + b^2(\sin^2\theta + \cos^2\theta)}$

$= \sqrt{a^2 + b^2}$

9. If the point P(*k*, 0) divides the line segment joining the points A(2, –2) and B(–7, 4) in the ratio 1 : 2, then the value of *k* is

(*a*) 1 (*b*) 2 (*c*) –2 (*d*) –1

Sol. (*d*),



$$k = \frac{(-7)\times 1 + 2\times 2}{1+2} = \frac{-7+4}{3} = -1$$

10. The value of *p*, for which the points A(3, 1), B(5, *p*) and C(7, –5) are collinear, is

(*a*) –2 (*b*) 2 (*c*) –1 (*d*) 1

Sol. (*a*), if points are collinear, then area of triangle formed by these points is 0.

$$\frac{1}{2}|3(p + 5) + 5(-5 - 1) + 7(1 - p)| = 0$$

$$\Rightarrow \frac{1}{2}|3p + 15 - 30 + 7 - 7p| = 0$$

$$\Rightarrow -4p - 8 = 0 \Rightarrow -4p = 8 \Rightarrow p = -2.$$

Question numbers **11** *to* **15,** *fill in the blanks. Each question is of* **1** *mark.*

11. In Fig. 1, ΔABC is circumscribing a circle, the length of BC is _______ cm.

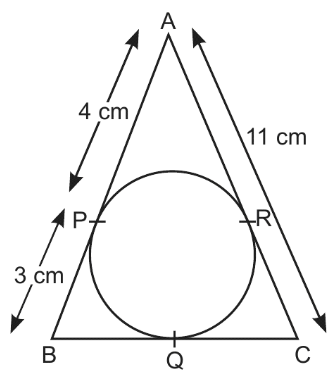


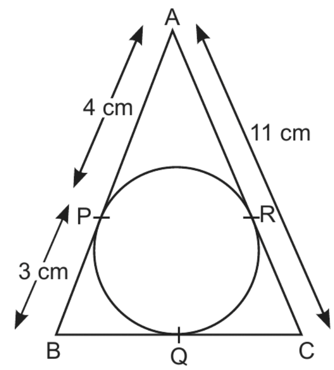
Fig.–1

Sol. $AR = AP = 4 \text{ cm}$

$\Rightarrow$ $CR = 11 \text{ cm} - 4 \text{ cm} = 7 \text{ cm}$

$BC = BQ + CQ = BP + CR$

$= 3 \text{ cm} + 7 \text{ cm} = 10 \text{ cm}$



12. Given $\Delta ABC \sim \Delta PQR$, if $\frac{AB}{PQ} = \frac{1}{3}$, then $\frac{ar(\Delta ABC)}{ar(\Delta PQR)} =$ ______

Sol. Since, $\Delta ABC \sim \Delta PQR$,

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \frac{BC^2}{QR^2} = \frac{AC^2}{PR^2}$$

[Ratio of the areas of two similar triangles is equal to the ratio of the square of their corresponding sides]

$$\Rightarrow \quad \frac{ar(\Delta ABC)}{ar(\Delta PQR)} = \frac{AB^2}{PQ^2} = \left(\frac{AB}{PQ}\right)^2 = \left(\frac{1}{3}\right)^2 = \frac{1}{9} \qquad \left[\text{Given, } \frac{AB}{PQ} = \frac{1}{3}\right]$$

13. ABC is an equilateral triangle of side $2a$, then length of one of its altitude is _____ .

Sol. $\sqrt{3}a$, Let AL is altitude of an equilateral ΔABC

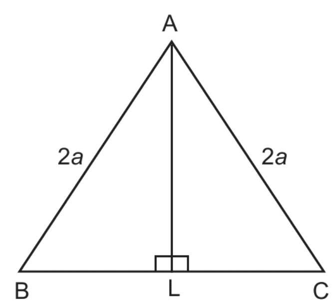
$\therefore$ $AL \perp BC$ and L is mid-point of BC.

In right-angled ΔALB

$$AB^2 = BL^2 + AL^2$$

$$\Rightarrow \quad (2a)^2 = (a)^2 + AL^2$$

$$\Rightarrow \quad AL^2 = 3a^2 \Rightarrow AL = \sqrt{3}a$$



14. $\frac{\cos 80°}{\sin 10°} + \cos 59° \operatorname{cosec} 31° =$ ________.

Sol. 2, as $\frac{\cos 80°}{\sin 10°} + \cos 59° \operatorname{cosec} 31° = \frac{\sin(90° - 80°)}{\sin 10°} + \cos 59° \sec(90° - 31°)$

$$= \frac{\sin 10°}{\sin 10°} + \cos 59° \cdot \sec 59° = 1 + 1 = 2.$$

15. The value of $\left(\sin^2\theta + \dfrac{1}{1+\tan^2\theta}\right) =$ ________.

OR

The value of $(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) =$ ________.

Sol.

$$\sin^2\theta + \frac{1}{1+\tan^2\theta} = \sin^2\theta + \frac{1}{\sec^2\theta}$$

$$= \sin^2\theta + \cos^2\theta = 1$$

OR

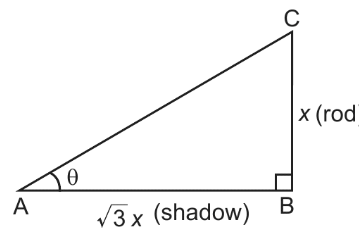
$$(1 + \tan^2\theta)(1 - \sin\theta)(1 + \sin\theta) = \sec^2\theta \cdot (1 - \sin^2\theta) = \sec^2\theta \cdot \cos^2\theta$$

$$= 1$$

Question numbers **16** *to* **20,** *are short answer type questions of* **1** *mark each.*

16. The ratio of the length of a vertical rod and the length of its shadow is $1 : \sqrt{3}$. Find the angle of elevation of the Sun at that moment.

Sol. Ratio is $1 : \sqrt{3}$



$$\tan\theta = \frac{x}{\sqrt{3}x} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $\theta = 30°$

17. Two cones have their heights in the ratio 1 : 3 and radii in the ratio 3 : 1. What is the ratio of their volumes?

Sol. Let height be x and $3x$

and radii be $3y$ and y

$$\therefore \quad \frac{V_1}{V_2} = \frac{\frac{1}{3}\pi(3y)^2 \cdot x}{\frac{1}{3}\pi(y)^2 \cdot 3x} = \frac{9}{3} = \frac{3}{1}$$

$$V_1 : V_2 = 3 : 1$$

18. A letter of English alphabet is chosen at random. What is the probability that the chosen letter is a consonant?

Sol. In English alphabets we have 21 consonants and 5 vowels.

$\therefore$ Probability of chosen letter is a consonant $= \dfrac{21}{26}$.

19. A die is thrown once. What is the probability of getting a number less than 3?

OR

If the probability of winning a game is 0.07, what is the probability of losing it?

Sol. Total outcomes = 6

Favourable outcomes = 2 (i.e. 1, 2)

$\therefore$ Probability of getting a number less than 3 = $\frac{2}{6} = \frac{1}{3}$

OR

Let A represents winning a game

$$P(A) = 0.07$$
$$P(\text{not } A) = P(\overline{A})$$
$$= 1 - P(A) = 1 - 0.07 = 0.93$$

20. If the mean of the first *n* natural number is 15, then find *n*.

Sol. Mean = 15, $\frac{1 + 2 + ... + n}{n} = 15$

$$\Rightarrow \quad \frac{1 + 2 + 3 + ... + n}{n} = 15$$
$$\Rightarrow \quad \frac{n(1 + n)}{2n} = 15$$
$$\Rightarrow \quad 1 + n = 30 \Rightarrow n = 29$$

SECTION – B

Question numbers **21** *to* **26** *carry* **2** *marks each.*

21. Show that $(a - b)^2$, $(a^2 + b^2)$ and $(a + b)^2$ are in AP.

Sol. If $(a - b)^2$, (a^2+b^2) and $(a + b)^2$ are in AP, then

$$(a^2 + b^2) - (a - b)^2 = (a + b)^2 - (a^2 + b^2)$$
$$\Rightarrow \quad a^2 + b^2 - a^2 - b^2 + 2ab = a^2 + b^2 + 2ab - a^2 - b^2$$
$$\Rightarrow \quad \text{L.H.S.} = \text{R.H.S.}$$

Hence, the result.

22. In Fig. 2, DE || AC and DC || AP. Prove that $\frac{BE}{EC} = \frac{BC}{CP}$.

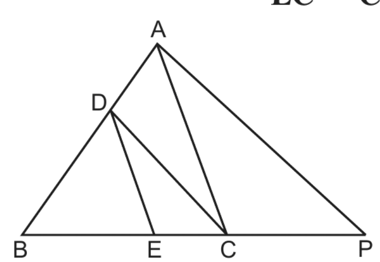


Fig.–2

OR

In Fig. 3, two tangents TP and TQ are drawn to a circle with centre O from an external point T. Prove that ∠PTQ = 2 ∠OPQ.

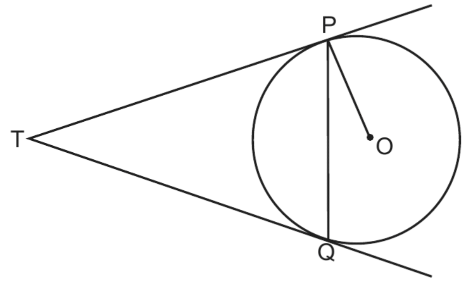
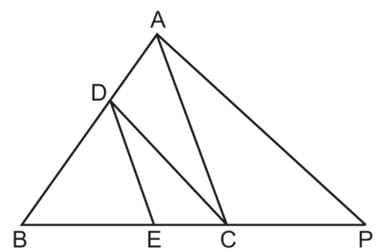


Fig.–3

Sol. In ΔABC, DE || AC



$$\Rightarrow \quad \frac{BD}{AD} = \frac{BE}{EC} \quad ...(i) \text{ [BPT Theorem]}$$

In ΔABP, DC || AP

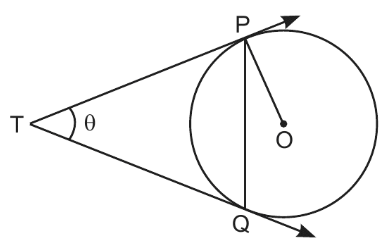
$$\Rightarrow \quad \frac{BD}{AD} = \frac{BC}{CP} \quad ...(ii) \text{ [BPT Theorem]}$$

From (*i*) and (*ii*)

$$\frac{BE}{EC} = \frac{BC}{CP}$$

OR

A circle with centre O.



An external point T from which TP and TQ are two tangents to the circle.

To prove: ∠PTQ = 2∠OPQ

Proof: Let ∠PTQ = θ

Now TP = TQ [Lengths of tangent segments from an external point to a circle are equal]

∴ ΔTPQ is an isosceles triangle

⇒ ∠TPQ = ∠TQP [Angles opposite to equal sides of a triangle are equal]

In triangle TPQ

$\angle PTQ + \angle TPQ + \angle TQP = 180°$ [Angle sum property of triangles]

$\Rightarrow \quad \theta + 2\angle TPQ = 180°$

$\Rightarrow \quad \angle TPQ = \frac{1}{2}(180° - \theta) = 90° - \frac{1}{2}\theta$

Also, $\angle OPT = 90°$ [Angle between tangent and radius through the point of contact]

So, $\angle OPQ = \angle OPT - \angle TPQ = 90° - \left(90° - \frac{1}{2}\theta\right) = \frac{1}{2}\theta$

$\Rightarrow \quad \angle OPQ = \frac{1}{2}\angle PTQ$

or $\angle PTQ = 2\angle OPQ$

23. The rod AC of a TV disc antenna is fixed at right angles to the wall AB and a rod CD is supporting the disc as shown in Fig. 4. If AC= 1.5 m long and CD = 3 m, find (*i*) tan θ (*ii*) sec θ + cosec θ.

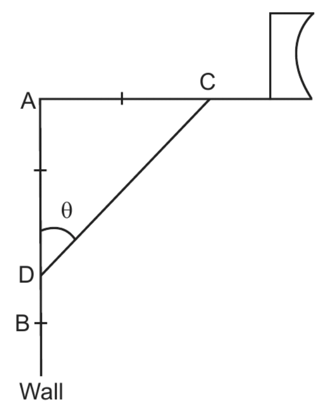
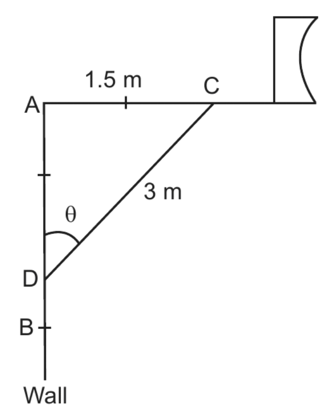


Fig.–4

Sol. AC = 1.5 m

CD = 3 m



$\therefore \quad \sin\theta = \frac{AC}{CD} = \frac{1.5}{3} = \frac{1}{2}$

$\Rightarrow \quad \theta = 30°$

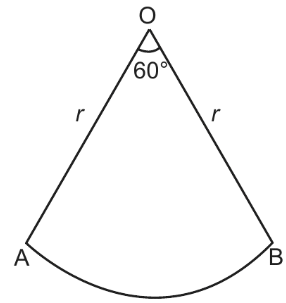
(*i*) $\tan\theta = \tan 30° = \frac{1}{\sqrt{3}}$

(*ii*) $\sec\theta + \text{cosec}\,\theta = \sec 30° + \text{cosec}\,30°$

$= \frac{2}{\sqrt{3}} + 2 = \frac{2}{\sqrt{3}}(1 + \sqrt{3})$

24. A piece of wire 22 cm long is bent into the form of an arc of a circle subtending an angle of 60° at its centre. Find the radius of the circle. $\left[\textbf{Use } \pi = \frac{22}{7}\right]$

Sol. Let the radius of the circle be r.



$$\widehat{AB} = 22$$

$$\frac{60}{360} \times 2\pi r = 22$$

$$\Rightarrow \quad \frac{1}{3} \times \frac{22}{7} \times r = 22 \Rightarrow r = 21 \text{ cm}$$

25. If a number x is chosen at random from the numbers –3, –2, –1, 0, 1, 2, 3. What is probability that $x^2 \leq 4$?

Sol. If $x^2 \leq 4$, then x can take values –2, –1, 0, 1, 2

$\therefore$ Probability of x such that $x^2 \leq 4 = \frac{5}{7}$

26. Find the mean of the following distribution:

Class	**3–5**	**5–7**	**7–9**	**9–11**	**11–13**
Frequency	**5**	**10**	**10**	**7**	**8**

OR

Find the mode of the following data:

Class	**0–20**	**20–40**	**40–60**	**60–80**	**80–100**	**100–120**	**120–140**
Frequency	**6**	**8**	**10**	**12**	**6**	**5**	**3**

Sol. Table for mean: let $A = 8$, here $h = 2$

C.I.	x	f	$d = \frac{x-8}{2}$	fd	fx
3 – 5	4	5	–2	–10	20
5 – 7	6	10	–1	–10	60
7 – 9	8	10	0	0	80
9 – 11	10	7	1	7	70
11 – 13	12	8	2	16	96
		40		3	326

$$\text{Mean} = A + \frac{\Sigma fd}{\Sigma f} \times h = 8 + \frac{3}{40} \times 2 = 8 + 0.15 = 8.15$$

Alt. method: $\quad \text{Mean} = \frac{\Sigma fx}{\Sigma f} = \frac{326}{40} = 8.15$

OR

Table for mode:

C.I.	f
0 – 20	6
20 – 40	8
40 – 60	10
60 – 80	12
80 – 100	6
100 – 120	5
120 – 140	3

Maximum frequency (f_0) = 12

Modal class is 60 – 80

$f_0 = 12, f_1 = 10, f_2 = 6, h = 20, l = 60$

← Modal class

$$\therefore \text{ Mode} = l + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$$

$$= 60 + \frac{12 - 10}{24 - 10 - 6} \times 20 = 60 + \frac{2}{8} \times 20 = 65$$

SECTION – C

Question numbers **27** *to* **34** *carry* **3** *marks each.*

27. Find a quadratic polynomial whose zeroes are reciprocals of the zeroes of the polynomial $f(x) = ax^2 + bx + c, a \neq 0, c \neq 0$.

OR

Divide the polynomial $f(x) = 3x^2 - x^3 - 3x + 5$ by the polynomial $g(x) = x - 1 - x^2$ and verify the division algorithm.

Sol. Given polynomial $f(x) = ax^2 + bx + c, a \neq 0, c \neq 0$

Let zeroes be α, β

$$\therefore \qquad \alpha + \beta = -\frac{b}{a}, \alpha\beta = \frac{c}{a} \qquad ...(i)$$

For required polynomial

Reciprocal zeroes are $\frac{1}{\alpha}, \frac{1}{\beta}$

$$\text{Sum} = \frac{1}{\alpha} + \frac{1}{\beta} = \frac{\beta + \alpha}{\alpha\beta} = \frac{-b/a}{c/a} = -\frac{b}{c} = -\frac{B}{A}$$

$$\text{Product} = \frac{1}{\alpha} \cdot \frac{1}{\beta} = \frac{1}{\alpha\beta} = \frac{1}{c/a} = \frac{a}{c} = \frac{C}{A}$$

Polynomial is $Ax^2 + Bx + C$

Here, $A = c, B = b, C = a$

i.e. $cx^2 + bx + a$

OR

$$
\begin{array}{r|l}
 & x - 2 \\
\hline
-x^2 + x - 1 & -x^3 + 3x^2 - 3x + 5 \\
 & -x^3 + x^2 - x \\
 & + \quad - \quad + \\
\hline
 & 2x^2 - 2x + 5 \\
 & 2x^2 - 2x + 2 \\
 & - \quad + \quad - \\
\hline
 & 3
\end{array}
$$

Polynomial (Dividend) : $-x^3 + 3x^2 - 3x + 5$

Divisor: $-x^2 + x - 1$

Quotient: $x - 2$

Remainder: 3

$$
\begin{aligned}
\text{Polynomial} &= \text{Quotient} \times \text{Divisor} + \text{Remainder} \\
&= (x - 2)(-x^2 + x - 1) + 3 \\
&= -x^3 + x^2 - x + 2x^2 - 2x + 2 + 3 \\
&= -x^3 + 3x^2 - 3x + 5
\end{aligned}
$$

Hence, verified.

28. Determine graphically the coordinates of the vertices of a triangle, the equations of whose sides are given by $2y - x = 8$, $5y - x = 14$ and $y - 2x = 1$.

OR

If 4 is a zero of the cubic polynomial $x^3 - 3x^2 - 10x + 24$, find its other two zeroes.

Sol. Consider equation $2y - x = 8$

$\Rightarrow$ $x = 2y - 8$

Some points on graph are

x	0	–8	–4
y	4	0	2

Consider equation $5y - x = 14$

$\Rightarrow$ $x = 5y - 14$

Some points on graph are

x	–4	1	6
y	2	3	4

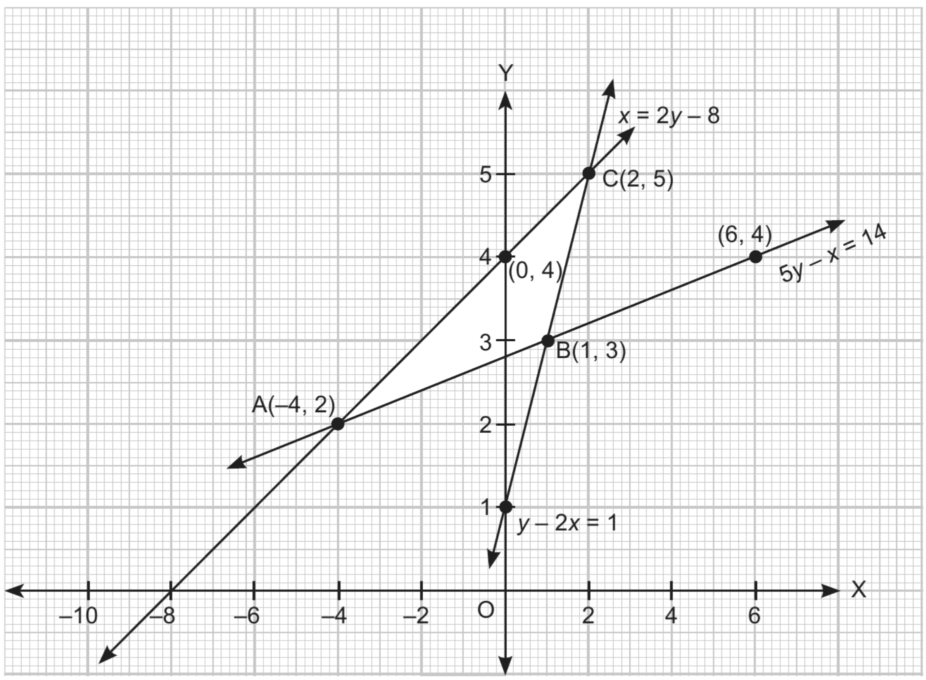
Consider equation $\qquad y - 2x = 1$

$\Rightarrow \qquad y = 2x + 1$

Some points on graph are

x	0	1	2
y	1	3	5

Plotting the points on graph we get triangle ABC with vertices A(–4. 2), B(1, 3), C(2, 5)



OR

$x = 4$ is a zero of polynomial $x^3 - 3x^2 - 10x + 24$

$\Rightarrow (x - 4)$ is a factor of $x^3 - 3x^2 - 10x + 24$

For other zeroes,

$$x^2 + x - 6 = 0$$

$$\Rightarrow \quad x^2 + 3x - 2x - 6 = 0$$

$$\Rightarrow \quad x(x + 3) - 2(x + 3) = 0$$

$$\Rightarrow \quad (x - 2)(x + 3) = 0$$

$$\Rightarrow \quad x - 2 = 0 \text{ or } x + 3 = 0$$

$$\Rightarrow \quad x = 2, -3$$

$\therefore$ Other two zeroes are 2, –3.

$$\begin{array}{r} x^2 + x - 6 \\ x - 4 \overline{)\, x^3 - 3x^2 - 10x + 24} \\ x^3 - 4x^2 \qquad\qquad \\ - \; + \qquad\qquad\quad \\ \hline x^2 - 10x + 24 \\ x^2 - 4x \qquad \\ - \;\; + \qquad\quad \\ \hline -6x + 24 \\ -6x + 24 \\ + \quad - \\ \hline 0 \end{array}$$

29. In a flight of 600 km, an aircraft was slowed due to bad weather. Its average speed for the trip was reduced by 200 km/hr and time of flight increased by 30 minutes. Find the original duration of flight.

Sol. Let original speed of the aircraft be x km/hr

Reduced speed = $(x - 200)$ km/hr

According to given condition

$$\frac{600}{x-200} - \frac{600}{x} = \frac{30}{60} = \frac{1}{2}$$

$$\frac{600x - 600x + 120000}{x(x-200)} = \frac{1}{2}$$

$$\Rightarrow \quad \frac{120000}{x^2 - 200x} = \frac{1}{2}$$

$$\Rightarrow \quad x^2 - 200x = 240000$$

$$\Rightarrow \quad x^2 - 200x - 240000 = 0$$

$$\Rightarrow \quad x^2 - 600x + 400x - 240000 = 0$$

$$\Rightarrow \quad x(x - 600) + 400(x - 600) = 0$$

$$\Rightarrow \quad (x + 400)(x - 600) = 0$$

$$\Rightarrow \quad x + 400 = 0 \text{ or } x - 600 = 0$$

$$\Rightarrow \quad x = -400 \text{ (rejected) or } x = 600$$

$\therefore$ original speed = 600 km/hr

$\therefore$ original duration of flight = $\frac{600}{600}$ = 1 hour

30. Find the area of triangle PQR formed by the points P(–5, 7), Q(–4, –5) and R(4, 5).

OR

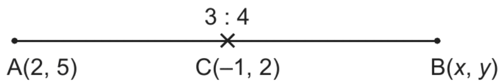
If the point C(–1, 2) divides internally the line segment joining A(2, 5) and B(*x*, *y*) in the ratio 3 : 4, find the coordinates of B.

Sol. Area of a triangle formed by the points P(–5, 7), Q(–4, –5) and R(4, 5) is

$$\frac{1}{2}|-5(-5-5) - 4(5-7) + 4(7+5)|$$

$$= \frac{1}{2}|50 + 8 + 48| = \frac{1}{2} \times 106 = 53 \text{ sq units.}$$

OR



C divides join of A (2, 5) and B (x, y) in the ratio 3 : 4

$\therefore$ Coordinates of C are $\left(\dfrac{3x+8}{3+4}, \dfrac{3y+20}{3+4}\right) = (-1, 2)$

$$\Rightarrow \qquad \left(\frac{3x+8}{7}, \frac{3y+20}{7}\right) = (-1, 2)$$

$$\Rightarrow \qquad \frac{3x+8}{7} = -1, \frac{3y+20}{7} = 2$$

$$\Rightarrow \qquad 3x + 8 = -7, 3y + 20 = 14$$

$$\Rightarrow \qquad 3x = -15, 3y = -6$$

$$\Rightarrow \qquad x = -5, y = -2$$

$\therefore$ Coordinates of B are (–5, –2)

31. In Fig. 5, $\angle D = \angle E$ and $\dfrac{AD}{DB} = \dfrac{AE}{EC}$, prove that BAC is an isosceles triangle.

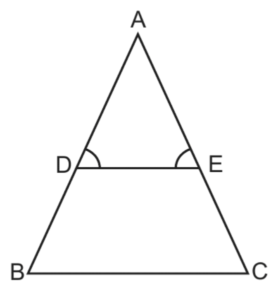
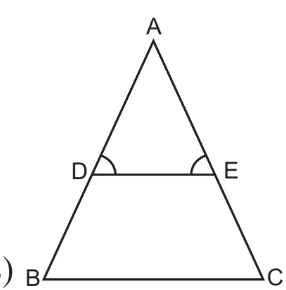


Fig.–5

Sol. **Given:** In given figure,

$$\angle D = \angle E \text{ and } \frac{AD}{DB} = \frac{AE}{EC}$$

To prove: BAC is an isosceles triangle.



Proof: In ΔADE, $\angle D = \angle E$ (given)

$\Rightarrow \qquad AE = AD \qquad ...(i)$ (sides opposites to equal angles)

Also, $\qquad \dfrac{AD}{DB} = \dfrac{AE}{EC} \qquad$ (given)

$\Rightarrow \qquad DB = EC$ ...(*ii*) [from (*i*)]

From (***i***) and (*ii*)

$AD + DB = AE + EC$

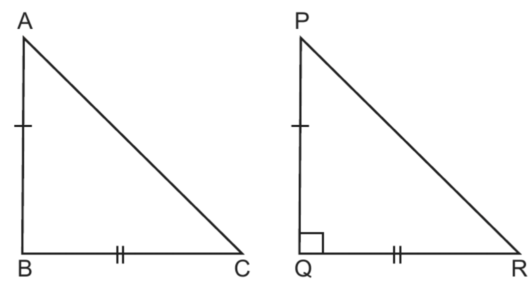
$\Rightarrow \qquad AB = AC$

$\Rightarrow$ BAC is an isosceles triangle.

32. In a triangle, if square of one side is equal to the sum of the squares of the other two sides, then prove that the angle opposite to the first side is a right angle.

Sol. Given: Triangle ABC, such that

$$AC^2 = AB^2 + BC^2$$



To prove: $\angle ABC = 90°$

Construction: Construct triangle PQR such that PQ = AB, QR = BC and $\angle PQR = 90°$

Proof: Triangle PQR is right angled at Q

$\therefore \qquad PR^2 = PQ^2 + QR^2$ (Pythagoras Theorem)

$\Rightarrow \qquad PR^2 = AB^2 + BC^2$ ...(*i*) ($\because$ AB = PQ, BC = QR construction)

Also $\qquad AC^2 = AB^2 + BC^2$ ...(*ii*) (given)

$\therefore \qquad PR^2 = AC^2$ [from (*i*) and (*ii*)]

$\Rightarrow \qquad PR = AC$

Consider triangles ABC and PQR

$AB = PQ$ (construction)

$BC = QR$ (construction)

$AC = PR$ (proved above)

$\therefore \qquad \Delta ABC \cong \Delta PQR$ (SSS)

$\Rightarrow \qquad \angle ABC = \angle PQR = 90°$

Hence $\angle ABC = 90°$

33. If $\sin\theta + \cos\theta = \sqrt{3}$, then prove that $\tan\theta + \cot\theta = 1$.

Sol. Consider $\sin\theta + \cos\theta = \sqrt{3}$

Squaring, we get

$$(\sin\theta + \cos\theta)^2 = (\sqrt{3})^2$$

$\Rightarrow \quad \sin^2\theta + \cos^2\theta + 2\sin\theta\cos\theta = 3$

$\Rightarrow \quad 1 + 2\sin\theta\cos\theta = 3$

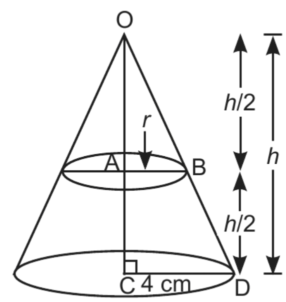
$\Rightarrow \quad 2\sin\theta\cos\theta = 2 \Rightarrow \sin\theta\cos\theta = 1 \quad$ ...(*i*)

Consider $\tan\theta + \cot\theta = \dfrac{\sin\theta}{\cos\theta} + \dfrac{\cos\theta}{\sin\theta}$

$$= \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \frac{1}{1} = 1 \text{ [from } (i)]$$

34. A cone of base radius 4 cm is divided into two parts by drawing a plane through the mid-point of its height and parallel to its base. Compare the volume of the two parts.

Sol. We have $\Delta OAB \sim \Delta OCD$



$\Rightarrow \quad \dfrac{OA}{OC} = \dfrac{r}{4}$

$\Rightarrow \quad \dfrac{h/2}{h} = \dfrac{r}{4} \Rightarrow r = 2$ cm

Volume of upper part (cone)

$r = 2$ cm, height $= \dfrac{h}{2}$ cm

$\therefore \quad V_1 = \dfrac{1}{3}\pi(2)^2 \cdot \dfrac{h}{2} = \dfrac{2}{3}\pi h \quad$ ...(*i*)

Volume of lower part (frustum)

$r = 2$ cm, $r_1 = 4$ cm, height $= \dfrac{h}{2}$ cm

$\therefore \quad V_2 = \dfrac{\pi \cdot \frac{h}{2}}{3}[4 + 16 + 2 \times 4]$

$$= \frac{\pi h}{6} \times 28 = \frac{14}{3}\pi h \quad \text{...}(ii)$$

$\therefore \quad \dfrac{V_1}{V_2} = \dfrac{\frac{2}{3}\pi h}{\frac{14}{3}\pi h} = \dfrac{1}{7} \quad$ [from (*i*) and (*ii*)]

$V_1 : V_2 = 1 : 7$

SECTION – D

Question numbers **35** *to* **40** *carry* **4** *marks each.*

35. Show that the square of any positive integer cannot be of the form $(5q + 2)$ or $(5q + 3)$ for any integer q.

OR

Prove that one of every three consecutive positive integers is divisible by 3.

Sol. Any integer with divisor 5 can be of the form $5n$, $5n + 1$, $5n + 2$, $5n + 3$, $5n + 4$

Consider $(5n)^2 = 25n^2 = 5(5n^2) = 5q, q = 5n^2 \in N$ ...(*i*)

$(5n + 1)^2 = 25n^2 + 10n + 1 = 5(5n^2 + 2n) + 1$

$= 5q + 1, q = 5n^2 + 2n \in N$ ...(*ii*)

$(5n + 2)^2 = 25n^2 + 20n + 4 = 5(5n^2 + 4n) + 4$

$= 5q + 4, q = 5n^2 + 4n \in N$ ...(*iii*)

$(5n + 3)^2 = 25n^2 + 30n + 9 = 5(5n^2 + 6n + 1) + 4$

$= 5q + 4, q = 5n^2 + 6n + 1 \in N$ ...(*iv*)

$(5n + 4)^2 = 25n^2 + 40n + 16 = 5(5n^2 + 8n + 3) + 1$

$= 5q + 1, q = 5n^2 + 8n + 3 \in N$ ...(*v*)

From (*i*) to (*v*) we notice square of any positive integer cannot be of the form $(5q + 2)$ or $(5q + 3)$ for any integer q.

OR

Let three consecutive positive integer be $n, n + 1, n + 2$

Case I: Let n is divisible by 3. Let $n = 3m$

then $n + 1 = 3m + 1$ is not divisible by 3

$n + 2 = 3m + 2$ is not divisible by 3

Case II: Let $n + 1$ is divisible by 3.

Let $n = 3m + 2$, which is not divisible by 3

$n + 1 = 3m + 2 + 1 = 3m + 3 = 3(m + 1)$

which is divisible by 3.

$n + 2 = 3m + 2 + 2 = 3m + 4 = 3(m + 1) + 1$

$= 3m' + 1$, which is not divisible by 3, $m' = m + 1 \in N$

Case III: Let $n + 2$ is divisible by 3.

Let $n = 3m + 1$, which is not divisible by 3

$n + 1 = 3m + 1 + 1 = 3m + 2$, which is not divisible by 3

$n + 2 = 3m + 1 + 2 = 3m + 3 = 3(m + 1) = 3m'$

$m' = m + 1$, which is divisible by 3.

Hence, from above three cases we notice one of every three consecutive positive integers is divisible by 3.

36. The sum of four consecutive numbers in AP is 32 and the ratio of the product of the first and last terms to the product of two middle terms is 7 : 15. Find the numbers.

OR

Solve : 1 + 4 + 7 + 10 + ... + x = 287

Sol. Refer to Sol. 12 page 130.

OR

Consider $1 + 4 + 7 + 10 + ... + x = 287$

Let $a_n = x, a = 1, d = 3$

$$S_n = 287 = \frac{n}{2}[2 \times 1 + (n-1)3]$$

$$\Rightarrow \quad 287 = \frac{n}{2}[2 + 3n - 3]$$

$$\Rightarrow \quad 574 = 3n^2 - n \Rightarrow 3n^2 - n - 574 = 0$$

$$\Rightarrow \quad 3n^2 - 42n + 41n - 574 = 0$$

$$\Rightarrow \quad 3n(n-14) + 41(n-14) = 0$$

$$\Rightarrow \quad (3n + 41)(n - 14) = 0$$

$$\Rightarrow \quad 3n + 41 = 0 \text{ or } n - 14 = 0$$

$$\Rightarrow \quad n = -\frac{41}{3}\text{(rejected) or } n = 14$$

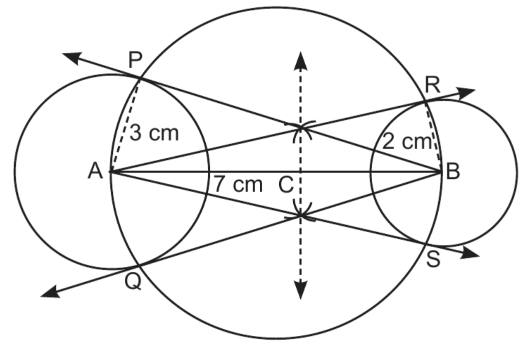
$$\therefore \quad a_{14} = x \Rightarrow 1 + (14-1)3 = x$$

$$\Rightarrow 1 + 39 = x \Rightarrow x = 40$$

37. Draw a line segment AB of length 7 cm. Taking A as centre, draw a circle of radius 3 cm and taking B as centre, draw another circle of radius 2 cm. Construct tangents to each circle from the centre of the other circle.

Sol. Steps of construction:

1. Draw AB = 7 cm. Taking A and B as centres, draw two circles of 3 cm and 2 cm radius.
2. Bisect line AB. Let mid-point of AB be C.
3. Taking C as centre, draw circle of AC radius which will intersect circles at P, Q, R, S. Join BP, BQ, AR, AS.
4. Required tangents are (*i*) BP and BQ (*ii*) AR and AS.



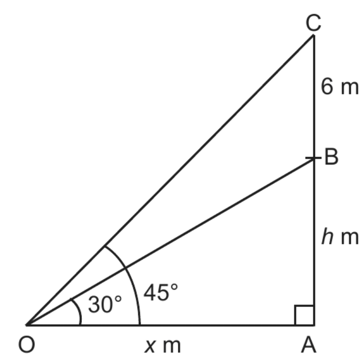
38. A vertical tower stands on a horizontal plane and is surmounted by a vertical flag-staff of height 6 m. At a point on the plane, the angle of elevation of the bottom and top of the flag-staff are 30° and 45° respectively. Find the height of the tower. (Take $\sqrt{3}$ = 1.73)

Sol. AB → Tower of height h m

BC → flag slide of height 6 m.

O is point of observation, $\angle AOB = 30°$, $\angle AOC = 45°$

Let OA = x m



In right-angled triangle OAB.

$$\frac{AB}{OA} = \tan 30°$$

$$\Rightarrow \quad \frac{h}{x} = \frac{1}{\sqrt{3}}$$

$$\Rightarrow \quad x = \sqrt{3}h \qquad ...(i)$$

In right-angled triangle OAC

$$\frac{AC}{OA} = \tan 45°$$

$$\Rightarrow \quad \frac{h+6}{x} = 1$$

$$\Rightarrow \quad h + 6 = x$$

$$\Rightarrow \quad h + 6 = \sqrt{3}h \qquad \text{[from } (i)\text{]}$$

$$\Rightarrow \quad \sqrt{3}h - h = 6 \Rightarrow (\sqrt{3} - 1)h = 6$$

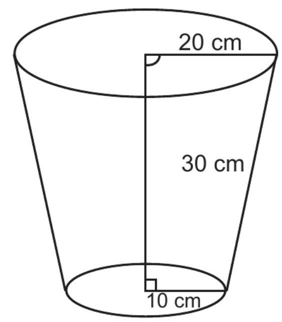
$$\Rightarrow \qquad h = \frac{6}{\sqrt{3}-1} = \frac{6(\sqrt{3}+1)}{3-1} = 3(\sqrt{3}+1)$$

$$\Rightarrow \qquad h = 3(1.73 + 1) = 3 \times 2.73 = 8.19 \text{ m}$$

$\therefore$ height of tower = 8.19 m.

39. A bucket in the form of a frustum of a cone of height 30 cm with radii of its lower and upper ends as 10 cm and 20 cm, respectively. Find the capacity of the bucket. Also find the cost of milk which can completely fill the bucket at the rate of ₹ 40 per litre. $\left(\text{Use } \pi = \frac{22}{7}\right)$

Sol. Let $r_1 = 20$ cm, $r_2 = 10$ cm and height, $h = 30$ cm



$$\text{Capacity of bucket} = \frac{\pi \times 30}{3}[(20)^2 + (10)^2 + 20 \times 10]$$

$$= 10\pi[400 + 100 + 200]$$

$$= 10 \times \frac{22}{7} \times 700 = 22000 \text{ cm}^3$$

$$1000 \text{ cm}^3 = 1\ l$$

$$\therefore \qquad \text{Capacity} = 22l$$

$$\text{Cost of milk} = ₹\ 40 \times 22 = ₹\ 880$$

40. The following table gives production yield per hectare (in quintals) of wheat of 100 farms of a village:

Production yield/hect.	40–45	45–50	50–55	55–60	60–65	65–70
No. of farms	4	6	16	20	30	24

Change the distribution to 'a more than' type distribution and draw its ogive.

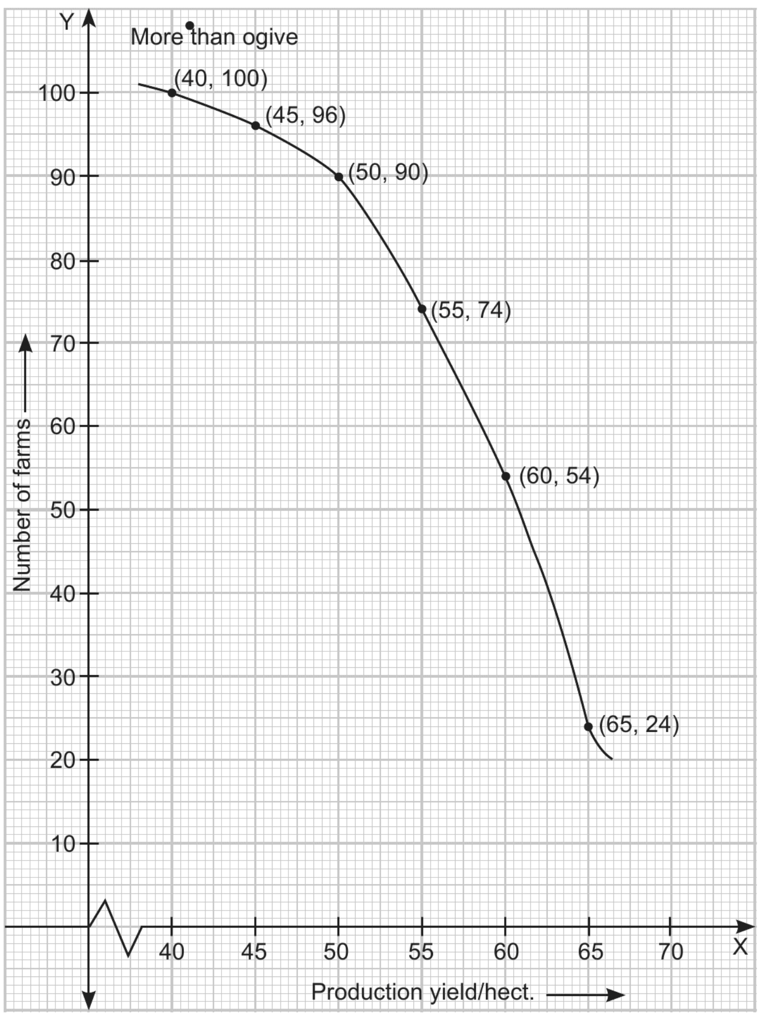
OR

The median of the following data is 525. Find the values of *x* and *y*, if total frequency is 100 :

Class	Frequency
0–100	2
100–200	5
200–300	*x*
300–400	12
400–500	17
500–600	20
600–700	*y*
700–800	9
800–900	7
900–1000	4

Sol. Table 'for more than' type ogive

Production yield/hect	Number of farms	Production yield/hect	No. of farms	Points
40–45	4	more than 40	100	(40, 100)
45–50	6	more than 45	96	(45, 96)
50–55	16	more than 50	90	(50, 90)
55–60	20	more than 55	74	(55, 74)
60–65	30	more than 60	54	(60, 54)
65–70	24	more than 65	24	(65, 24)



OR

Table for median = 525

C.I.	f	$c.f$	
0–100	2	2	
100–200	5	7	
200–300	x	$7 + x$	
300–400	12	$19 + x$	
400–500	17	$36 + x$	
500–600	20	$56 + x$	← Median class
600–700	y	$56 + x + y$	
700–800	9	$65 + x + y$	
800–900	7	$72 + x + y$	
900–1000	4	$76 + x + y$	
	100		

We have $76 + x + y = 100$

$\Rightarrow$ $x + y = 24$ ...(*i*)

As median is 525, therefore median class is 500 – 600.

$$l = 500, \frac{N}{2} = \frac{100}{2} = 50, c = 36 + x, f = 20, h = 100$$

$$\text{Median} = l + \frac{\frac{N}{2} - c}{f} \times h$$

$$525 = 500 + \frac{50 - (36 + x)}{20} \times 100$$

$$25 = (50 - 36 - x)5$$

$$25 = 70 - 5x$$

$\Rightarrow$ $5x = 45 \Rightarrow x = 9$

from (*i*)

$$9 + y = 24 \Rightarrow y = 15$$

$\therefore\ x = 9, y = 15$

SET-2 [UNCOMMON QUESTIONS TO SET 1]

14. $\left(\frac{\sin 35°}{\cos 55°}\right)^2 + \left(\frac{\cos 43°}{\sin 47°}\right)^2 - 2\cos 60° =$ **________.**

Sol. Consider $\left(\frac{\sin 35°}{\cos 55°}\right)^2 + \left(\frac{\cos 43°}{\sin 47°}\right)^2 - 2\cos 60°$

$$= \left(\frac{\cos (90° - 35°)}{\cos 55°}\right)^2 + \left(\frac{\sin (90° - 43°)}{\sin 47°}\right)^2 - 2\cos 60°$$

$$= \left(\frac{\cos 55°}{\cos 55°}\right)^2 + \left(\frac{\sin 47°}{\sin 47°}\right)^2 - 2\cos 60°$$

$$= (1)^2 + (1)^2 - 2 \times \frac{1}{2}$$

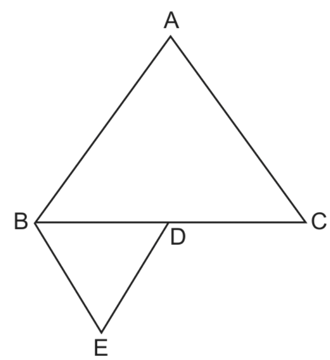
$$= 1 + 1 - 1 = 1$$

15. ABC and BDE are two equilateral triangles such that D is the mid-point of BC. Ratio of the areas of triangles ABC and BDE is _______.

Sol. 4 : 1, as

$$BD = \frac{1}{2}BC$$

$$\Delta ABC \sim \Delta EBD \text{ (AAA)}$$



$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta EBD)} = \frac{BC^2}{BD^2} = \frac{(2BD)^2}{BD^2}$$

$$= \frac{4BD^2}{BD^2} = \frac{4}{1}$$

$$ar(\Delta ABC) : ar(\Delta BDE) = 4 : 1$$

20. A die is thrown once. What is the probability of getting an even prime number?

Sol. Total outcomes for a throw of a die = 6.

i.e. numbers 1, 2, 3, 4, 5, 6

Favourable outcomes for getting an even prime number = 1

i.e. number 2

$\therefore$ Probability of getting an even prime number $= \frac{1}{6}$

25. Find the sum of first 20 terms of the following AP : 1, 4, 7, l0, ...

Sol. AP is 1, 4, 7, 10, ...

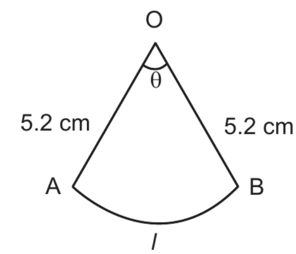
$$a = 1, d = 3$$

$$S_{20} = \frac{20}{2}[2 \times 1 + (20 - 1)3] = 10[2 + 57]$$

$$= 590$$

26. The perimeter of a sector of a circle of radius 5.2 cm is 16.4 cm. Find the area of the sector.

Sol. We have



$$OA + \widehat{AB} + OB = 16.4$$

$$5.2 + l + 5.2 = 16.4$$

$$l = 6.0 \text{ cm}$$

$$\Rightarrow \quad \frac{\theta}{360} \times 2\pi r = 6 \Rightarrow \frac{\theta\pi r}{360} = 3 \quad ...(i)$$

$$\text{Area of sector} = \frac{\theta}{360}\pi r^2 = \frac{\theta\pi r}{360} \times r$$

$$= 3 \times 5.2 \quad \text{[from } (i)\text{]}$$

$$= 15.6 \text{ cm}^2$$

32. A train covers a distance of 480 km at a unifoml speed. If the speed had been 8 km/hr less, then it would have taken 3 hours more to cover the same distance. Find the original speed of the train.

Sol. Let original speed of train be x km/hr

Now, new speed be $= (x - 8)$ km/hr

$$\therefore \quad \frac{480}{x-8} - \frac{480}{x} = 3$$

$$\frac{480x - 480x + 3840}{x(x-8)} = 3$$

$$\Rightarrow \quad 3x(x - 8) = 3840 \Rightarrow x^2 - 8x - 1280 = 0$$

$$\Rightarrow \quad x^2 - 40x + 32x - 1280 = 0$$

$$\Rightarrow \quad x(x - 40) + 32(x - 40) = 0$$

$\Rightarrow \quad (x + 32)(x - 40) = 0$

$\Rightarrow \quad x + 32 = 0$ or $x - 40 = 0$

$\Rightarrow \quad x = -32$(rejected) or $x = 40$

$\therefore$ original speed of train = 40 km/hr

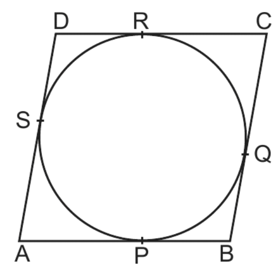
33. Prove that the parallelogram circumscribing a circle is a rhombus.

Sol. **Given:** ABCD is parallelogram circumscribing a circle.

To prove: ABCD is a rhombus

Proof: We have, DR = DS ...(*i*)

[Lengths of tangents drawn from an external point to a circle are equal]



Also, AP = AS ...(*ii*)

BP = BQ ...(*iii*)

CR = CQ ...(*iv*)

Adding (*i*), (*ii*), (*iii*) and (*iv*),

(DR + CR) + (AP + BP) = (DS + AS) + (BQ + CQ)

$\Rightarrow$ CD + AB = AD + BC

$\Rightarrow$ 2AB = 2AD [$\because$ In parallelogram, opposite sides are equal AB = CD and AD = BC]

$\Rightarrow$ AB = AD

$\therefore$ AB = AD = BC = CD

Hence, ABCD is a rhombus as all sides are equal in rhombus.

34. Prove that $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$.

Sol. Consider $2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1$

$= 2\{(\sin^2\theta)^3 + (\cos^2\theta)^3\} - 3\{(\sin^2\theta)^2 + (\cos^2\theta)^2\} + 1$

Using $a^3 + b^3 = (a + b)^3 - 3ab(a + b)$ and $a^2 + b^2 = (a + b)^2 - 2ab$, we get

$= 2[\{\sin^2\theta + \cos^2\theta\}^3 - 3\sin^2\theta\cos^2\theta(\sin^2\theta + \cos^2\theta)\}]$

$-3[\{(\sin^2\theta + \cos^2\theta)^2 - 2\sin^2\theta\cos^2\theta\}] + 1$

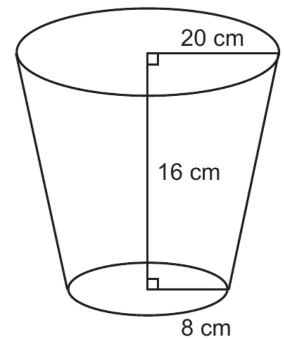
$= 2[(1)^3 - 3\sin^2\theta\cos^2\theta(1)] - 3[(1)^2 - 2\sin^2\theta\cos^2\theta] + 1$

$= 2 - 6\sin^2\theta\cos^2\theta - 3 + 6\sin^2\theta\cos^2\theta + 1$

$= 0$

39. A bucket is in the form of a frustum of a cone of height 16 cm with radii of its lower and upper circular ends as 8 cm and 20 cm respectively. Find the cost of milk which can completely fill the bucket, at the rate of ₹ 40 per litre. (Use $\pi = 3.14$)

Sol. Let $r_1 = 8$ cm, $r_2 = 20$ cm and $h = 16$ cm



$$\text{Volume of milk} = \frac{\pi \times 16}{3}[(8)^2 + (20)^2 + 8 \times 20]$$

$$= \frac{\pi \times 16}{3}[64 + 400 + 160]$$

$$= \frac{16\pi}{3}[624] = 16\pi \times 208$$

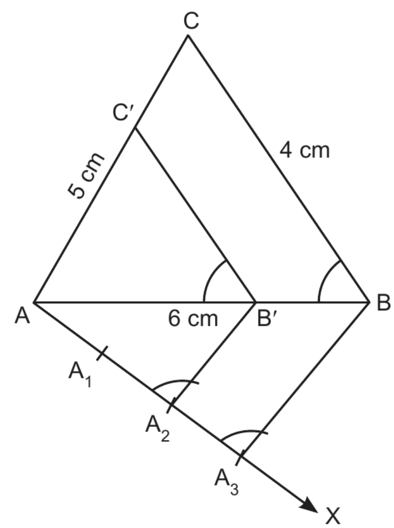
$$= 3328 \times 3.14 = 10449.92 \text{ cm}^3$$

$$= 10.44992\, l = 10.45\, l \text{ (app.)}$$

$\therefore$ cost of milk = ₹ 40 × 10.45 = ₹ 418

40. Construct a triangle with sides 4 cm, 5 cm and 6 cm. Then construct another triangle whose sides are $\frac{2}{3}$ times the corresponding sides of the first triangle.

Sol. Steps of construction:



(*i*) A ΔABC with sides AB = 6 cm, BC = 4 cm, AC = 5 cm is drawn

(*ii*) AX is drawn below AB
and points A_1, A_2, A_3 are taken on AX such that $AA_1 = A_1A_2 = A_2A_3$

(*iii*) A_3 and B are joined

(*iv*) A_2B' is drawn parallel to A_3B, meeting AB at B′.

(*v*) B′C′ is drawn parallel to BC meeting AC at C′.

Then, AB′C′ is required triangle.

SET-3 [UNCOMMON QUESTIONS TO SET 1 AND SET 2]

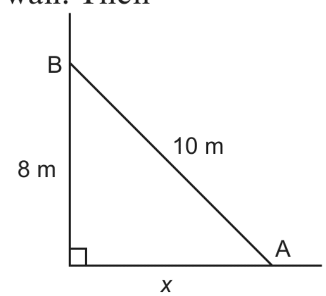
14. A ladder 10 m long reaches a window 8 m above the ground. The distance of the foot of the ladder from the base of the wall is ________ m.

Sol. 6, let the foot of the ladder is x m away from the base of the wall. Then

$$x^2 + (8)^2 = (10)^2$$

$\Rightarrow$ $$x^2 = 100 - 64 = 36$$

$\Rightarrow$ $$x = 6 \text{ m}$$



15. $\dfrac{2\cos 67°}{\sin 23°} - \dfrac{\tan 40°}{\cot 50°} - \cos 0° =$ **________.**

Sol. Consider $\dfrac{2\cos 67°}{\sin 23°} - \dfrac{\tan 40°}{\cot 50°} - \cos 0°$

$$= \frac{2\sin(90° - 67°)}{\sin 23°} - \frac{\cot(90° - 40°)}{\cot 50°} - \cos 0°$$

$$= \frac{2\sin 23°}{\sin 23°} - \frac{\cot 50°}{\cot 50°} - 1$$

$$= 2 - 1 - 1 = 0$$

20. A pair of dice is thrown once. What is the probability of getting a doublet?

Sol. Possible outcomes are

{(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)
(2, 1), (2, 2), (2, 3), (2, 4), (2, 5), (2, 6)
(3, 1), (3, 2), (3, 3), (3, 4), (3, 5), (3, 6)
(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6)
(5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6)
(6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

Total elementary events = 36

Outcomes of doublet = {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)} i.e. 6

$$\text{P(getting a doublet)} = \frac{6}{36} = \frac{1}{6}$$

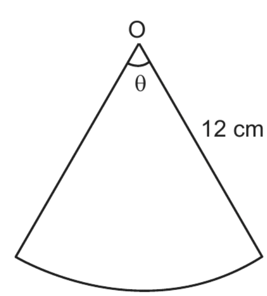
25. The minute hand of a clock is 12 cm long. Find the area of the face of the clock described by the minute hand in 35 minutes.

Sol. Angle θ formed in 35 minutes = 35 × 6 = 210°

$\therefore$ $$\text{Area} = \frac{210°}{360°} \times \pi(12)^2$$

$$= \frac{7}{12} \times \frac{22}{7} \times 144 \text{ cm}^2$$

$$= 264 \text{ cm}^2$$



26. The sum of the first 7 terms of an AP is 63 and that of its next 7 terms is 161. Find the AP.

Sol. Let a be first term, d be common difference of an AP

$$S_7 = 63 \text{ and } S_{14} = S_7 + 161 = 63 + 161 = 224$$

$$\frac{7}{2}[2a + 6d] = 63 \Rightarrow a + 3d = 9 \quad ...(i)$$

$$\frac{14}{2}[2a + 13d] = 224 \Rightarrow 2a + 13d = 32 \quad ...(ii)$$

Solving (i) and (ii), we get

$$a = 3, d = 2$$

$\therefore$ AP is 3, 5, 7, 9, ...

32. A man can row a boat downstream 20 km in 2 hours and upstream 4 km in 2 hours. Find his speed of rowing in still water. Also find the speed of the stream.

Sol. Let speed of the boat in still water be x km/hr and speed of stream be y km/hr

$\therefore$ Speed upstream $= (x - y)$ km/hr

Speed downstream $= (x + y)$ km/hr

According to given condition,

$$\frac{20}{x + y} = 2 \Rightarrow x + y = 10 \quad ...(i)$$

and $$\frac{4}{x - y} = 2 \Rightarrow x - y = 2 \quad ...(ii)$$

Solving (i) and (ii), we get

$$x = 6, y = 4$$

$\therefore$ Speed of boat in still water = 6 km/hr

Speed of stream = 4 km/hr

33. In given Fig. 5, two circles touch each other at the point C. Prove that the common tangent to the circles at C, bisects the common tangent at P and Q.

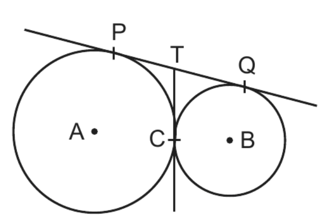


Fig.–5

Sol. PT and TC are tangents to circle with centre A.

$\therefore$ PT = TC [tangent from common point T] ...(*i*)

Similarly TQ and TC are tangents to circle with centre B

$\therefore$ TQ = TC ...(*ii*)

From (*i*) and (*ii*), PT = TQ

$\therefore$ CT bisects PQ.

34. Prove that: $\frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1} = \frac{1+\cos\theta}{\sin\theta}$.

Sol. Consider

$$\text{LHS} = \frac{\cot\theta + \operatorname{cosec}\theta - 1}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{(\cot\theta + \operatorname{cosec}\theta) - (\operatorname{cosec}^2\theta - \cot^2\theta)}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{(\cot\theta + \operatorname{cosec}\theta) - (\operatorname{cosec}\theta + \cot\theta)(\operatorname{cosec}\theta - \cot\theta)}{\cot\theta - \operatorname{cosec}\theta + 1}$$

$$= \frac{(\cot\theta + \operatorname{cosec}\theta)(1 - \operatorname{cosec}\theta + \cot\theta)}{(\cot\theta - \operatorname{cosec}\theta + 1)}$$

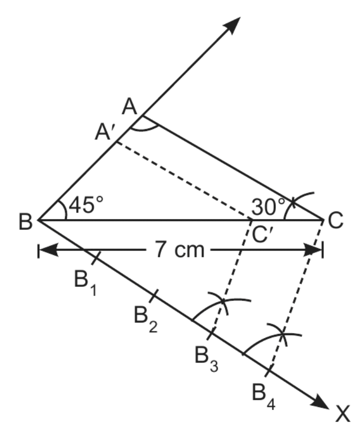
$$= \cot\theta + \operatorname{cosec}\theta = \frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}$$

$$= \frac{\cos\theta + 1}{\sin\theta} = \frac{1+\cos\theta}{\sin\theta} = \text{R.H.S.}$$

39. Draw a ΔABC with BC = 7 cm, $\angle$B = 45° and $\angle$A = 105°. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of ΔABC.

Sol. In ΔABC,

$$\angle A + \angle B + \angle C = 180°$$
$$105° + 45° + \angle C = 180°$$
$$150° + \angle C = 180°$$
$$\angle C = 30°$$



Steps of Construction:

(*a*) Draw BC = 7 cm.

(*b*) At B draw $\angle$B = 45° and $\angle$C = 30°.

(*c*) Draw acute angle $\angle$CBX, below BC at B.

(*d*) Mark B_1, B_2, B_3, B_4 on BX such that

$BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

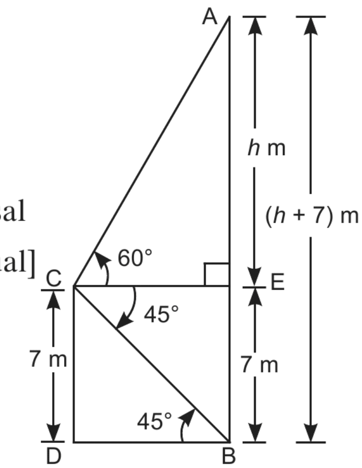
(*e*) Join B_4C, draw $B_3C' \parallel B_4C$, where C′ is a point on BC.

(*f*) Draw $C'A' \parallel AC$, where A′ is a point on BA.

(*g*) $\Delta A'BC'$ is the required triangle.

40. From the top of a 7 m high building the angle of elevation of the top of a tower is 60° and the angle of depression of its foot is 45°. Determine the height of the tower.

Sol. Let height of the tower AB be 'H' m



$\therefore \quad H = AE + EB \Rightarrow H = (h + 7)\text{m}$

Given height of building CD = 7 m

$\angle ACE = 60°, \angle ECB = 45°$

$\Rightarrow \quad \angle CBD = 45°$ [$\because$ CE || DB, CB is transversal $\therefore$ alternate pair of angles are equal]

Now in right-triangle CDB,

$$\frac{CD}{DB} = \tan 45° \Rightarrow \frac{7}{DB} = 1$$

$\Rightarrow \quad DB = 7$ m

In right triangle AEC,

$$\frac{AE}{EC} = \tan 60°$$

$$\Rightarrow \quad \frac{h}{7} = \sqrt{3} \qquad [\because DB = EC = 7 \text{ m}]$$

$$\Rightarrow \quad h = 7\sqrt{3}\,\text{m}$$

Now, $\quad H = AB = h + 7 = 7\sqrt{3} + 7 = 7(\sqrt{3} + 1)$ m

Hence, height of the tower is $7(\sqrt{3}+1)$m

EXAMINATION PAPERS_2019

[DELHI (SET - I, II, III)]

Time Allowed: 3 Hours] *[Maximum Marks: 80*

General Instructions:

(i) All questions are compulsory.

*(ii) This question paper consists of **30** questions divided into four sections – A, B, C and D.*

*(iii) Section A contains **6** questions of **1** mark each, Section B contains **6** questions of **2** marks each, Section C contains **10** questions of **3** marks each and Section D contains **8** questions of **4** marks each.*

(iv) There is no overall choice. However, an internal choice has been provided in two questions of 1 mark, two questions of 2 marks, four questions of 3 marks each and three questions of 4 marks each. You have to attempt only one of the alternatives in all such questions.

(v) Use of calculator is not permitted.

SET-I

SECTION – A

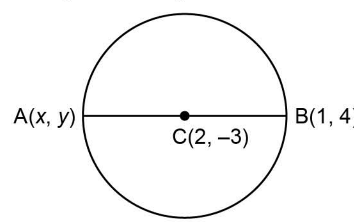
Question numbers **1** *to* **6** *carry* **1** *mark each.*

1. Find the coordinates of a point A, where AB is diameter of a circle whose centre is (2, –3) and B is the point (1, 4).

Sol. AB is diameter of the circle.

Let C be centre of circle, coordinates of C are (2, –3). Here, C is mid-point of AB (diameter).

Let coordinates of A are (x, y).



$\therefore \quad \frac{x+1}{2} = 2$ and $\frac{y+4}{2} = -3$

$\Rightarrow \quad x + 1 = 4$ and $y + 4 = -6$

$\Rightarrow \quad x = 3$ and $y = -10$

$\therefore$ The coordinates of A are (3, – 10).

2. For what values of k, the roots of the equation $x^2 + 4x + k = 0$ are real?

Or

Find the value of k for which the roots of the equation $3x^2 - 10x + k = 0$ are reciprocal of each other.

Sol. The given equation is $x^2 + 4x + k = 0$

$$D = b^2 - 4ac$$

$\Rightarrow \quad D = (4)^2 - 4 \times 1 \times k$

For real roots $D \geq 0$

$\Rightarrow \quad D = 16 - 4k \geq 0$

$\Rightarrow \quad 16 - 4k \geq 0$

$\Rightarrow \quad 16 \geq 4k \Rightarrow k \leq 4$

Or

Given equation is $3x^2 - 10x + k = 0$

Let one root of the equation is α and another root is $\frac{1}{\alpha}$

Product of the roots $= \alpha \cdot \frac{1}{\alpha}$

So $$\alpha \cdot \frac{1}{\alpha} = \frac{k}{3}$$

$\Rightarrow$ $$1 = \frac{k}{3} \quad \Rightarrow \quad k = 3.$$

3. Find A if tan 2A = cot(A – 24°)

Or

Find the value of ($\sin^2 33° + \sin^2 57°$)

Sol.
$$\tan 2A = \cot (A - 24°)$$
$$\cot(90° - 2A) = \cot(A - 24°)$$

On equating both sides

$$90° - 2A = A - 24°$$
$$90° + 24° = 3A$$
$$114° = 3A \quad \Rightarrow \quad A = 38°$$

Or

$$\sin^2 33° + \sin^2 57° = \sin^2(90° - 57°) + \sin^2 57°$$
$$= \cos^2 57° + \sin^2 57° \qquad [\text{Using } \sin(90° - \theta) = \cos \theta]$$
$$= 1.$$

4. How many two digits numbers are divisible by 3?

Sol. Two digits numbers divisible by 3 are 12, 15 99

Here, $a = 12, d = 3, a_n = 99$

Let two digits numbers divisible by 3 be n

So, $$a_n = a + (n - 1) d$$

$\Rightarrow$ $$99 = 12 + (n - 1) 3$$

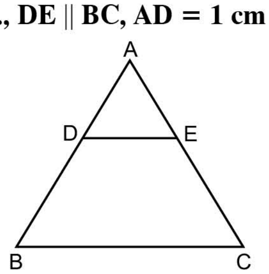
$\Rightarrow$ $$99 = 9 + 3n$$

$\Rightarrow$ $$3n = 90$$

$\Rightarrow$ $$n = 30$$

30, two digits numbers are divisible by 3.

5. In fig., DE || BC, AD = 1 cm and BD = 2 cm. What is the ratio of the ar(ΔABC) to the ar(ΔADE)?

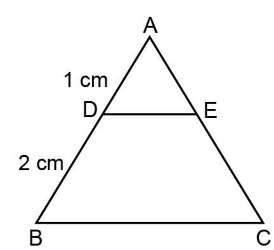


Sol. Given: In ΔABC, DE || BC

Also AD = 1 cm and BD = 2 cm

To find: ar(ΔABC) : ar(ΔADE)

As DE | | BC (Given)



$\angle D = \angle B$ and $\angle E = \angle C$(Corresponding angles)

$\therefore$ $\Delta ADE \sim \Delta ABC$ (By AA similarity)

$$\therefore \quad \frac{ar(\Delta ABC)}{ar(\Delta ADE)} = \frac{AB^2}{AD^2} = \left(\frac{3}{1}\right)^2 = \frac{9}{1}.$$

$ar(\Delta ABC) : ar(\Delta ADE) = 9 : 1.$

6. Find a rational number between $\sqrt{2}$ and $\sqrt{3}$.

Sol. Let p be rational number between $\sqrt{2}$ and $\sqrt{3}$

So, $\sqrt{2} < p < \sqrt{3}$

On squaring throughout, we have

$2 < p^2 < 3$

The perfect squares which lie between 2 and 3 are 2.25, 2.56, 2.89.

We have, $2 < 2.25 < 2.56 < 2.89 < 3$

Taking square root throughout

$\sqrt{2} < 1.5 < 1.6 < 1.7 < \sqrt{3}$

The rational numbers between $\sqrt{2}$ and $\sqrt{3}$ are 1.5, 1.6, 1.7 and more.

SECTION – B

Question numbers **7** *to* **12** *carry* **2** *marks each.*

7. Find the HCF of 1260 and 7344 using Euclid's algorithm.

Or

Show that every positive odd integer is of the form $4q + 1$ or $4q + 3$ where q is some integer.

Sol. By Euclid's algorithm

We have
$$7344 = 5(1260) + 1044$$
$$1260 = 1(1044) + 216$$
$$1044 = 4(216) + 180$$
$$216 = 1(180) + 36$$
$$180 = 5(36) + 0$$

HCF of 7344 and 1260 is 36.

Or

Let N be any positive integer and $b = 4$

Then by Euclid's division lemma, $N = 4q + r$,

$$0 \leq r < 4, q > 0$$

$\therefore$ $N = 4q$ or $4q + 1$ or $4q + 2$ or $4q + 3$

(*i*) when $N = 4q = 2(2q)$ = even

(*ii*) when $N = 4q + 1 = 2(2q) + 1$ = odd

(*iii*) when $N = 4q + 2 = 2(2q + 1)$ = even

(*iv*) when $N = 4q + 3 = 2(2q + 1) + 1$ = odd

$\therefore$ When $N = 4q + 1$ or $4q + 3$, then it is odd

$\Rightarrow$ Any positive odd integer is of the form $4q + 1$ or $4q + 3$.

8. Which term of the AP: 3, 15, 27, 39, . . . will be 120 more than its 21st term?

Or

If S_n the sum of first n terms of an AP is given by $S_n = 3n^2 - 4n$, find the nth term.

Sol. Let nth term of the A.P. 3, 15, 27, 39, ... is 120 more than its 21st term

$$a_n = a_{21} + 120$$

$$\Rightarrow \quad a + (n-1)d = a + 20d + 120 \quad \text{(where } a \text{ is } 1^{st} \text{ term and } d \text{ is common difference)}$$

$$\Rightarrow \quad (n-1)d = 20d + 120 \quad \ldots(i)$$

Here, $d = 15 - 3 = 12$

Put in eq. (i), we get

$$(n-1)12 = 20(12) + 120$$

$$12n - 12 = 240 + 120$$

$$12n = 360 + 12,$$

$$n = \frac{372}{12} = 31$$

31^{st} term is 120 more than its 21^{st} term.

Or

Sum of 1st n terms is $S_n = 3n^2 - 4n$

n^{th} term a_n is given by $a_n = S_n - S_{n-1}$

$$= 3n^2 - 4n - [3(n-1)^2 - 4(n-1)]$$

$$= 3n^2 - 4n - [3(n^2 + 1 - 2n) - 4n + 4]$$

$$= 3n^2 - 4n - [3n^2 - 6n + 3 - 4n + 4]$$

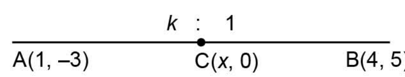
$$= 3n^2 - 4n - 3n^2 + 6n - 3 + 4n - 4$$

$$= 6n - 7$$

Hence, $a_n = 6n - 7$.

9. Find the ratio in which the segment joining the points (1, –3) and (4, 5) is divided by x-axis? Also find the coordinates of this point on x-axis.

Sol. Let x-axis divides AB in the ratio $k : 1$ at point C(x, 0) where coordinates of A and B are (1, –3) and (4, 5).



$$\therefore \quad x = \frac{4k+1}{k+1}$$

and $$0 = \frac{5k-3}{k+1} \Rightarrow 5k = 3 \Rightarrow k = \frac{3}{5}$$

$\therefore$ C(x, 0) divides AB in the ratio 3 : 5

also $$x = \frac{4\left(\frac{3}{5}\right)+1}{\frac{3}{5}+1} = \frac{12+5}{3+5} = \frac{17}{8}$$

So, coordinates of C are $\left(\frac{17}{8}, 0\right)$.

10. A game consists of tossing a coin 3 times and noting the outcome each time. If getting the same result in all the tosses is a success, find the probability of losing the game.

Sol. A coin is tossed 3 times. Number of possible outcomes are

$\begin{Bmatrix} \text{HHH} & \text{HTH} & \text{THH} & \text{HHT} \\ \text{TTT} & \text{THT} & \text{HTT} & \text{TTH} \end{Bmatrix}$. Total possible outcomes = 8

Favourable outcomes of getting same result in all the tosses HHH and TTT.

Number of favourable outcomes = 2

Probability of success = $\frac{2}{8} = \frac{1}{4}$. So, probability of losing = $1 - \frac{1}{4} = \frac{3}{4}$

11. A die is thrown once. Find the probability of getting a number which

(*i*) is a prime number

(*ii*) lies between 2 and 6.

Sol. A die is thrown, number of possible outcomes = 6

Prime numbers are 2, 3 and 5.

(*i*) Probability of getting a prime number = $\frac{3}{6} = \frac{1}{2}$ (Prime numbers are 2, 3, 5)

(*ii*) Probability of getting a number lies between 2 and 6 = $\frac{3}{6} = \frac{1}{2}$.

12. Find c if the system of equations $cx + 3y + (3 - c) = 0$; $12x + cy - c = 0$ has infinitely many solutions?

Sol. The system of equations are

$$cx + 3y + (3 - c) = 0 \quad ...(i)$$

$$12x + cy - c = 0 \quad ...(ii)$$

For infinitely many solutions, we have

$$\frac{c}{12} = \frac{3}{c} = \frac{3-c}{-c}$$

$\Rightarrow$ $c^2 = 36$ and $-3c = 3c - c^2$

$\Rightarrow$ $c = \pm 6$ and $c^2 = 6c$

$\Rightarrow$ $c = \pm 6$ and $c = 0, 6$

Hence, possible value is $c = 6$.

SECTION – C

Question numbers **13** *to* **22** *carry* **3** *marks each.*

13. Prove that $\sqrt{2}$ is an irrational number.

Sol. Let us assume that $\sqrt{2}$ is rational.

$\therefore$ There exists coprime integers a and b $(b \neq 0)$

Such that

$$\sqrt{2} = \frac{a}{b} \quad \Rightarrow \sqrt{2}b = a$$

Squaring on both sides, we get

$$2b^2 = a^2 \quad ...(i)$$

$\Rightarrow$ 2 divides a^2 $\Rightarrow$ 2 divides a

So, we can write

$$a = 2c \text{ for some integer } c \quad ...(ii)$$

From (*i*) and (*ii*),

$$2b^2 = 4c^2 \Rightarrow b^2 = 2c^2$$

$\Rightarrow$ 2 divides b^2 $\Rightarrow$ 2 divides b

$\therefore$ 2 is a common factor of a and b.

But this contradicts the fact that a and b are coprimes.

This contradiction has arisen because of our incorrect assumption that $\sqrt{2}$ is rational.

Hence, $\sqrt{2}$ is irrational.

14. Find the value of k such that the polynomial $x^2 - (k + 6)x + 2(2k - 1)$ has sum of its zeros equal to half of their product.

Sol. The given polynomial is $x^2 - (k + 6)x + 2(2k - 1)$

Let α and β be the zeros of polynomial

so $$\alpha + \beta = -\left[\frac{-(k+6)}{1}\right] = k + 6$$

$$\alpha\beta = \frac{2(2k-1)}{1} = 4k - 2$$

So $$\alpha + \beta = \frac{1}{2}\alpha\beta$$

$\Rightarrow$ $$k + 6 = \frac{1}{2}(4k - 2)$$

$$2k + 12 = 4k - 2$$

$$14 = 2k \quad \Rightarrow \quad k = 7.$$

15. A father's age is three times the sum of the ages of his two children. After 5 years his age will be two times the sum of their ages. Find the present age of the father.

Or

A fraction becomes $\frac{1}{3}$ when 2 is subtracted from the numerator and it becomes $\frac{1}{2}$ when 1 is subtracted from the denominator. Find the fraction.

Sol. Let sum of the ages of two children is x.

Age of father $= y$

As per condition $$y = 3x$$

$\Rightarrow$ $$3x - y = 0 \qquad ...(i)$$

Also, $$y + 5 = 2(x + 10)$$

$$y + 5 = 2x + 20$$

$$2x - y = -15 \qquad ...(ii)$$

Eq. (*i*) – eq. (*ii*)

We have $$x = 15$$

$$\text{age of father} = 3 \times 15 = 45 \text{ years.}$$

Or

Let the numerator be x

denominator be y

as per condition (1st condition)

$$\frac{x-2}{y} = \frac{1}{3} \quad \Rightarrow \quad 3x - 6 = y$$

$\Rightarrow \qquad 3x - y = 6 \qquad ....(i)$

(2nd condition)

$$\frac{x}{y-1} = \frac{1}{2}$$

$$2x = y - 1$$

$\Rightarrow \qquad 2x - y = -1 \qquad ....(ii)$

eq. (i) – eq. (ii)

$x = 7$ and $y = 2x + 1 \Rightarrow y = 2 \times 7 + 1 \Rightarrow y = 14 + 1 = 15$

Fraction is $\frac{7}{15}$.

16. Find the point on y-axis which is equidistant from the points (5, – 2) and (–3, 2).

Or

The line segment joining the points A(2, 1) and B(5, – 8) is trisected at the points P and Q such that P is nearer to A. If P also lies on the line given by $2x - y + k = 0$, find the value of k.

Sol. Let point on y-axis be $(0, a)$

Now, distance of this point from (5, –2) is equal to distance from point (–3, 2)

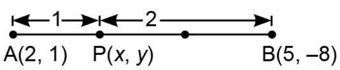
i.e., $\sqrt{5^2 + (-2-a)^2} = \sqrt{(3)^2 + (a-2)^2}$

Squaring and simplifying, we get

$$25 + 4 + a^2 + 4a = 9 + a^2 + 4 - 4a$$

$\Rightarrow \quad 8a = -16 \quad \Rightarrow \quad a = -2$

Or



Since point P trisects AB, then PA : PB = 1 : 2. Coordinates of P are

$$x = \frac{5 \times 1 + 2 \times 2}{1+2} = \frac{5+4}{3} = 3$$

and $$y = \frac{-8 \times 1 + 1 \times 2}{1+2} = \frac{-8+2}{3} = -2$$

Now, P lies on $2x - y + k = 0$

On putting values of x and y, we get

$2 \times 3 + 2 + \text{k} = 0 \Rightarrow 6 + 2 + k = 0 \Rightarrow k = -8$.

17. Prove that $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2 = 7 + \tan^2\theta + \cot^2\theta$.

Or

Prove that (1 + cot A – cosec A)(1 + tan A + sec A) = 2.

Sol. Taking LHS $(\sin\theta + \text{cosec}\,\theta)^2 + (\cos\theta + \sec\theta)^2$

$$= \sin^2\theta + \text{cosec}^2\theta + 2 + \cos^2\theta + \sec^2\theta + 2$$

$$= \sin^2\theta + \cos^2\theta + \text{cosec}^2\theta + \sec^2\theta + 4$$

$$= 1 + 1 + \cot^2\theta + 1 + \tan^2\theta + 4$$

$$= 7 + \cot^2\theta + \tan^2\theta \qquad \text{(R.H.S.)}$$

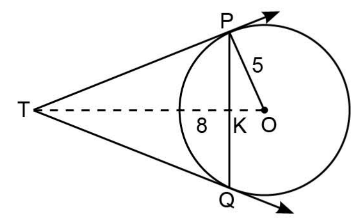
Or

(1 + cot A – cosec A)(1 + tan A + sec A) = 2

Taking LHS

$$(1 + \cot A - \operatorname{cosec} A)(1 + \tan A + \sec A)$$
$$= \left(1+\frac{\cos A}{\sin A}-\frac{1}{\sin A}\right)\left(1+\frac{\sin A}{\cos A}+\frac{1}{\cos A}\right)$$
$$= \left(\frac{\sin A+\cos A-1}{\sin A}\right)\left(\frac{\cos A+\sin A+1}{\cos A}\right)$$
$$= \frac{(\sin A+\cos A)^2-1}{\sin A\cos A} = \frac{\sin^2 A+\cos^2 A+2\sin A\cos A-1}{\sin A\cos A}$$
$$= \frac{1+2\sin A\cos A-1}{\sin A\cos A} = \frac{2\sin A\cos A}{\sin A\cos A} = 2.$$

18. In Fig. PQ is a chord of length 8 cm of a circle of radius 5 cm and centre O. The tangents at P and Q intersect at point T. Find the length of TP.



Sol. Given: PQ = 8 cm, OP = 5 cm

PQ is chord of circle, OP is radius and O is centre of circle.

To find: TP

Construction: Join OP

OT is perpendicular bisector of PQ

$\therefore$ $$PK = \frac{1}{2}PQ = \frac{1}{2} \times 8 = 4 \text{ cm}$$

In ΔPOK,

$$OP^2 = OK^2 + PK^2$$
$$(5)^2 = x^2 + (4)^2$$
$$\Rightarrow 25 = x^2 + 16$$
$$\Rightarrow x^2 = 25 - 16 = 9$$
$$\Rightarrow x = 3 \text{ cm.}$$

(i.e. OK = 3 cm)

In ΔOPT $$OT^2 = PT^2 + OP^2$$
$$(OK + TK)^2 = PT^2 + (5)^2$$
$$(3 + TK)^2 = PT^2 + 25$$
$$9 + TK^2 + 6TK = PT^2 + 25 \qquad ...(i)$$

In ΔPTK $$TP^2 = PK^2 + TK^2$$
$$\Rightarrow TK^2 = TP^2 - PK^2 \qquad ...(ii)$$

Put in eq. (*i*), we have

$$9 + TP^2 - PK^2 + 6\,TK = TP^2 + 25$$
$$9 + TP^2 - (4)^2 + 6\,TK = PT^2 + 25$$
$$\Rightarrow 9 + 6TK = 41$$
$$6TK = 32$$
$$TK = \frac{16}{3}$$

Put TK in eq (*ii*), we have

$$\frac{256}{9} = TP^2 - (4)^2$$

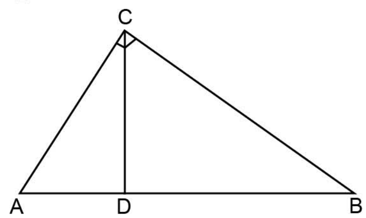
$$\Rightarrow \quad \frac{256}{9} + 16 = TP^2$$

$$\Rightarrow \quad \frac{256+144}{9} = TP^2$$

$$\Rightarrow \quad \frac{400}{9} = TP^2$$

$$\Rightarrow \quad PT = \sqrt{\frac{400}{9}} = \frac{20}{3} \text{ cm.}$$

19. In Fig., $\angle ACB = 90°$ and $CD \perp AB$, prove that $CD^2 = BD \times AD$.



Or

If P and Q are the points on sides CA and CB respectively of ΔABC, right angled at C, prove that $(AQ^2 + BP^2) = (AB^2 + PQ^2)$.

Sol. Given: $\angle ACB = 90°$, $CD \perp AB$

To prove: $CD^2 = BD \times AD$

Proof: In ΔADC and ΔACB

$$\angle A = \angle A \quad \text{(common)}$$

$$\angle ADC = \angle ACB = 90° \quad \text{(given)}$$

$\therefore$ $\Delta ADC \sim \Delta ACB$ (by AA)

$$\Rightarrow \quad \frac{AD}{CD} = \frac{AC}{BC} \quad ...(i)$$

In ΔBCA and ΔBDC

$$\angle CBA = \angle CBD \quad \text{(common)}$$

and $\angle BCA = \angle BDC = 90°$

$\therefore$ $\Delta BCA \sim \Delta BDC$ (by AA)

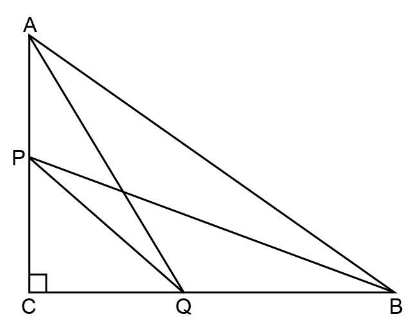
$$\Rightarrow \quad \frac{AC}{BC} = \frac{CD}{BD} \quad ...(ii)$$

Equating (*i*) and (*ii*)

$$\frac{AD}{CD} = \frac{CD}{BD}$$

$$\Rightarrow \quad CD^2 = BD \times AD.$$

Or



Given: P and Q are the points on sides CA and CB

To prove: $(AQ^2 + BP^2) = (AB^2 + PQ^2)$

Construction: Join PQ

Proof: In ΔACB, $\angle C = 90°$ by Pythagoras theorem

$$AB^2 = AC^2 + BC^2 \qquad ...(i)$$

In ΔAQC, $\angle C = 90°$

$\therefore$ By Pythagoras theorem

$$AQ^2 = AC^2 + CQ^2 \qquad ...(ii)$$

In ΔBPC, $\angle C = 90°$

$$BP^2 = CP^2 + BC^2 \qquad ...(iii)$$

In ΔPCQ, $\angle C = 90°$

$$\Rightarrow \quad PQ^2 = PC^2 + CQ^2 \qquad ...(iv)$$

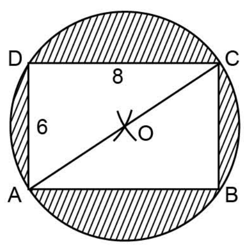
Adding (*ii*) and (*iii*), we have

$$AQ^2 + BP^2 = AC^2 + CQ^2 + CP^2 + BC^2$$
$$= AC^2 + BC^2 + CQ^2 + CP^2$$

From (*i*) and (*iv*), we get

$$AQ^2 + BP^2 = AB^2 + PQ^2.$$

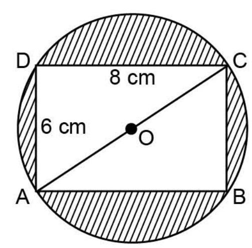
20. Find the area of the shaded region of Fig., if ABCD is a rectangle with sides 8 cm and 6 cm and O is the centre of circle. (Take π = 3.14)



Sol. O is centre of circle, AB is diameter

$\therefore$ In ΔADC, $\angle D = 90°$

$$\therefore \quad AC^2 = AD^2 + CD^2 = (6)^2 + (8)^2 = 36 + 64$$
$$AC^2 = 100 \quad \Rightarrow \quad AC = \sqrt{100} = 10 \text{ cm}$$
$$OC = \text{radius of circle} = \frac{1}{2} AC = \frac{1}{2} \times 10 = 5 \text{ cm}$$
$$\text{Area of circle} = \pi r^2 = 3.14 \times (5)^2 = 25 \times 3.14 = 78.5$$
$$\text{Area of rectangle} = 6 \times 8 = 48 \text{ cm}^2$$
$$\text{Area of shaded region} = \text{area of circle} - \text{area of rectangle}$$
$$= 78.5 - 48 = 30.5 \text{ cm}^2.$$



21. Water in a canal, 6 m wide and 1.5 m deep, is flowing with a speed of 10 km/h. How much area will it irrigate in 30 minutes, if 8 cm standing water is needed?

Sol. Width of canal = 6 m, depth = 1.5 m

Speed of flowing water = 10 km/h

Volume of the water flowing in 30 minutes or $\frac{1}{2}$hr = 6m × 1.5 m × 10 × 30

$$= \frac{6 \times 15 \times 10 \times 1000 \times 30}{10 \times 60} \text{ m}^3 = 45000 \text{ m}^3$$

8 cm of standing water is required.

Let the required area be A

$\therefore$ Volume of water required $= A\left(\frac{8}{100}\right) m^3$

According to the question,

$$45000 = \frac{A \times 8}{100}$$

$$\Rightarrow \quad \frac{45000 \times 100}{8} = A$$

$$\Rightarrow \quad A = 562500 \text{ m}^2.$$

22. Find the mode of the following frequency distribution.

Class	**0-10**	**10-20**	**20-30**	**30-40**	**40-50**	**50-60**	**60-70**
Frequency	**8**	**10**	**10**	**16**	**12**	**6**	**7**

Sol.

Classes	f
0 — 10	8
10 — 20	10
20 — 30	10 $\rightarrow f_0$
30 — 40	16 $\rightarrow f_1$
40 — 50	12 $\rightarrow f_2$
50 — 60	6
60 — 70	7

$$\text{Mode} = l + \left(\frac{f_1 - f_0}{2f_1 - f_0 - f_2}\right) \times h$$

$$= 30 + \left[\frac{16 - 10}{32 - 10 - 12}\right] \times 10$$

$$= 30 + \frac{6}{10} \times 10 = 30 + 6 = 36$$

SECTION – D

Question numbers **23** *to* **30** *carry* **4** *marks each.*

23. Two water taps together can fill a tank in $1\frac{7}{8}$ hours. The tap with longer diameter takes 2 hours less than the tap with smaller one to fill the tank separately. Find the time in which each tap can fill the tank separately.

Or

A boat goes 30 km upstream and 44 km downstream in 10 hours. In 13 hours, it can go 40 km upstream and 55 km downstream. Determine the speed of the stream and that of the boat in still water.

Sol. Let time taken by tap with longer diameter = x hours

time taken by tap with smaller diameter = $(x + 2)$ hours

As per condition

$$\frac{1}{x} + \frac{1}{x+2} = \frac{8}{15}$$

$$\frac{x + 2 + x}{x(x+2)} = \frac{8}{15}$$

$$\frac{2x + 2}{x(x+2)} = \frac{8}{15}$$

$$15(x + 1) = 4x(x + 2)$$

$$15(x+1) = 4x^2 + 8x$$
$$4x^2 + 8x - 15x - 15 = 0$$
$$4x^2 - 7x - 15 = 0$$
$$4x^2 - 12x + 5x - 15 = 0$$
$$4x(x-3) + 5(x-3) = 0$$
$$(x-3)(4x+5) = 0$$

$\Rightarrow \quad x = 3, x = -\frac{5}{4}$ (not possible)

Tap with longer diameter fill the tank in 3 hours.

Tap with smaller diameter fill the tank in 5 hours.

Or

Let speed of stream $= y$

speed of boat $= x$

during upstream speed $= x - y$

during downstream speed $= x + y$

As per condition

Ist condition $\quad \frac{30}{x-y} + \frac{44}{x+y} = 10$

$\therefore \quad 30A + 44B = 10 \quad$ ...(*i*) [where $\frac{1}{x-y} = A$ and $\frac{1}{x+y} = B$]

2nd condition

$$\frac{40}{x-y} + \frac{55}{x+y} = 13$$

$40A + 55B = 13 \quad$ ...(*ii*)

Solving (*i*) and (*ii*) for A and B

$30A + 44B = 10\,] \times 4$

$40A + 55B = 13\,] \times 3$

$120A + 176B = 40$

$120A + 165B = 39$

$- \quad - \quad -$

$11B = 1 \quad \Rightarrow \quad B = \frac{1}{11} \quad \Rightarrow \quad \frac{1}{x+y} = \frac{1}{11}$

$\Rightarrow x + y = 11 \quad$ ...(*iii*)

From eq (*i*), we have

$30A + 44\left(\frac{1}{11}\right) = 10$

$30A + 4 = 10 \quad \Rightarrow \quad A = \frac{6}{30} = \frac{1}{5}$

$\Rightarrow \quad \frac{1}{x-y} = \frac{1}{5} \Rightarrow x - y = 5 \quad$ ..(*iv*)

Adding (*iii*) and (*iv*)

$2x = 16 \quad \Rightarrow \quad x = 8$ km/hour

$\Rightarrow \quad y = 3$ km/hour

24. If the sum of first four terms of an AP is 40 and that of first 14 terms is 280. Find the sum of its first *n* terms.

Sol. Let first term of an AP is a and common difference is d.

$$\therefore \quad S_4 = 40$$

$$\frac{4}{2}[2a + (4-1)d] = 40$$

$$\Rightarrow \quad \frac{4}{2}[2a + 3d] = 40$$

$$2a + 3d = 20 \quad \text{...(i)}$$

$$S_{14} = 280$$

$$\frac{14}{2}[2a + (14-1)d] = 280$$

$$\Rightarrow \quad \frac{14}{2}[2a + 13d] = 280$$

$$2a + 13d = 40 \quad \text{...(ii)}$$

eq. (*ii*) – eq. (*i*)

$$2a + 13d = 40$$

$$2a + 3d = 20$$

$$- \quad - \quad -$$

$$10d = 20 \quad \Rightarrow \quad d = 2$$

Put $d = 2$ in eq. (*i*), we have

$$2a + 3(2) = 20$$

$$2a = 14 \quad \Rightarrow \quad a = 7$$

$$\text{Sum of } n \text{ terms} = \frac{n}{2}[14 + (n-1)2]$$

$$= \frac{n}{2}[2n + 12] = n(n + 6)$$

25. Prove that $\dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1} = \dfrac{1}{\sec A - \tan A}$.

Sol. LHS $= \dfrac{\sin A - \cos A + 1}{\sin A + \cos A - 1}$

Dividing numerator and denominator by cos A, we have

$$= \frac{\tan A - 1 + \sec A}{\tan A + 1 - \sec A}$$

$$= \frac{\tan A + \sec A - (\sec^2 A - \tan^2 A)}{\tan A - \sec A + 1}$$

$$= \frac{(\tan A + \sec A)[1 - \sec A + \tan A]}{(\tan A - \sec A + 1)}$$

$$= \tan A + \sec A = \frac{(\tan A + \sec A)(\sec A - \tan A)}{(\sec A - \tan A)}$$

$$= \frac{\sec^2 A - \tan^2 A}{\sec A - \tan A}$$

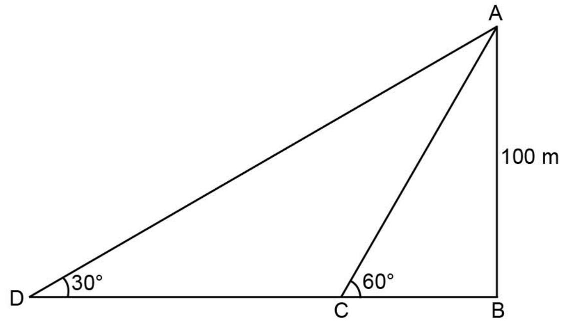
$$= \frac{1}{\sec A - \tan A}$$

26. A man in a boat rowing away from a light house 100 m high takes 2 minutes to change the angle of elevation of the top of the light house from 60° to 30°. Find the speed of the boat in metres per minute. [Use $\sqrt{3} = 1.732$]

Or

Two poles of equal heights are standing opposite to each other on either side of the road, which is 80 m wide. From a point between them on the road, the angles of elevation of the top of the poles are 60° and 30° respectively. Find the height of the poles and the distances of the point from the poles.

Sol.



AB is light house of height 100 m

The boat is at C making an angle of elevation 60° with the top of light house.

After 2 minutes, boat is at point D, making angle 30° with the top of light house.

Let $\quad BC = y \text{ m and } CD = x \text{ m}$

In ΔABC, $\quad \tan 60° = \frac{100}{y}$

$$\sqrt{3} = \frac{100}{y}$$

$$y = \frac{100}{\sqrt{3}} \quad \text{...}(i)$$

In ΔABD,

$$\tan 30° = \frac{100}{x+y}$$

$$\frac{1}{\sqrt{3}} = \frac{100}{x+y}$$

$$x + y = 100\sqrt{3} \quad \text{...}(ii)$$

$\therefore$
$$x = 100\sqrt{3} - y$$

$$= 100\sqrt{3} - \frac{100}{\sqrt{3}} = \frac{100(3-1)}{\sqrt{3}} = \frac{200}{\sqrt{3}} \text{ m}$$

$$\text{speed of boat} = \frac{\text{distance}}{\text{time}} = \frac{\frac{200}{\sqrt{3}}}{2}$$

$$= \frac{200}{\sqrt{3}} \times \frac{1}{2}$$

$$= \frac{100}{\sqrt{3}} \text{ m/minute}$$

$$= \frac{100}{1.732} = 57.73 \text{ m/minute}$$

Or

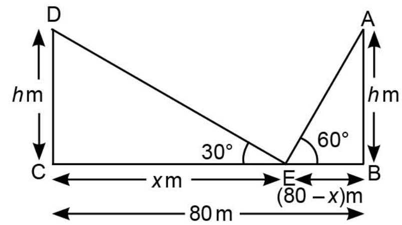
Let AB and CD are two poles of equal heights h m.

Let $AB = CD = h$ m [Height of the poles]

Given: BC = 80 m [Width of the road]

Let $CE = x$ m

$\therefore$ $BE = (80 - x)$ m



In ΔCDE, $\frac{CD}{CE} = \frac{h}{x} = \tan 30°$

$$\frac{h}{x} = \frac{1}{\sqrt{3}} \Rightarrow x = \sqrt{3}h \quad ... (i)$$

In ΔABE, $\frac{AB}{BE} = \tan 60° \Rightarrow \frac{h}{80 - x} = \sqrt{3}$

$\Rightarrow$ $h = 80\sqrt{3} - \sqrt{3}x$

$\Rightarrow$ $\sqrt{3}x = 80\sqrt{3} - h$

$\Rightarrow$ $x = \frac{80\sqrt{3} - h}{\sqrt{3}}$ ... (ii)

From equation (i) and (ii), we get

$$\sqrt{3}h = \frac{80\sqrt{3} - h}{\sqrt{3}}$$

$\Rightarrow$ $3h = 80\sqrt{3} - h$

$\Rightarrow$ $4h = 80\sqrt{3} \Rightarrow h = 20\sqrt{3}$

Substituting h in equation (i),

$$x = h\sqrt{3} = 20\sqrt{3} \times \sqrt{3} = 60 \text{ m}$$

Hence, position of the point is at a distance of 60 m from pole CD and 20 m from pole AB.

27. Construct a ΔABC in which CA = 6 cm, AB = 5 cm and $\angle BAC = 45°$. Then construct a triangle whose sides are $\frac{3}{5}$ of the corresponding sides of ΔABC.

Sol. Steps of construction:

(i) Draw AC = 6 cm.

(ii) At A, draw angle 45°; mark it $\angle YAC = 45°$

(iii) Cut AY by an arc of 5 cm from A and mark this point B, such that AB = 5 cm.

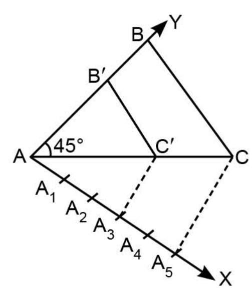
(iv) Draw an acute angle $\angle CAX$ below AC at point A.

(v) Draw 5 equal marks A_1, A_2, A_3, A_4, A_5 such that $AA_1 = A_1A_2 = A_2A_3 = A_3A_4 = A_4A_5$

(vi) Join A_5C. Then draw $A_3C' \parallel A_5C$ where C′ is a point on AC.

(vii) At C′, draw $C'B' \parallel CB$.

$\Delta B'AC'$ is the required triangle.



28. A bucket open at the top is in the form of a frustum of a cone with a capacity of 12308.8 cm³. The radii of the top and bottom of circular ends of the bucket are 20 cm and 12 cm respectively. Find the height of the bucket and also the area of the metal sheet used in making it. (Use $\pi = 3.14$)

Sol. Radii of the top of the bucket = 20 cm = R

Radii of the bottom of the bucket = 12 cm = r

$$\text{Capacity of the bucket} = \frac{\pi h}{3}[R^2 + r^2 + Rr]$$

$$12308.8 = \frac{3.14}{3} \times h[400 + 144 + 240]$$

$$= \frac{3.14}{3} \times h[784]$$

$$h = \frac{12308.8 \times 3}{3.14 \times 784} = 15 \text{ cm}$$

$$\text{Slant height of frustum} = l = \sqrt{h^2 + (R - r)^2}$$

$$= \sqrt{(15)^2 + (20 - 12)^2} = \sqrt{225 + 64}$$

$$= \sqrt{289} = 17 \text{ cm}$$

Metal sheet used = curved surface area of bucket + area of bottom of bucket

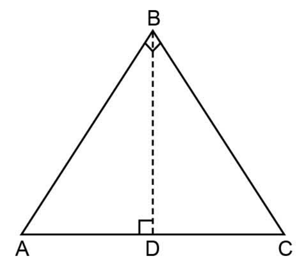
$$= \pi l(R + r) + \pi r^2$$

$$= 3.14 \times 17(20 + 12) + 3.14 \times (12)^2$$

$$= 1708.16 + 452.16 = 2160.32 \text{ cm}^2$$

29. Prove that in a right angle triangle, the square of the hypotenuse is equal to the sum of squares of the other two sides.

Sol. Given ΔABC is a right-angled triangle



where $\angle B = 90°$

To prove: $AC^2 = AB^2 + BC^2$

Construction: draw $BD \perp AC$

Proof: In ΔADB and ΔABC

$$\angle A = \angle A \quad \text{(common)}$$

$$\angle ADB = \angle ABC = 90°$$

$\therefore$ $\Delta ADB \sim \Delta ABC$ (by AA)

$$\frac{AD}{AB} = \frac{AB}{AC}$$

$\Rightarrow$ $AB^2 = AD \times AC$ ...(*i*)

Similarly ΔCBA and ΔCDB

$$\angle CBA = \angle CDB = 90°$$

$$\angle ACB = \angle BCD \quad \text{(common)}$$

$\Rightarrow$ $\dfrac{CB}{CD} = \dfrac{AC}{BC}$

$BC^2 = AC \times CD$ ...(*ii*)

Adding (*i*) and (*ii*)

$$AB^2 + BC^2 = AD \times AC + AC \times CD$$

$$= AC(AD + DC) = AC \times AC$$

$\Rightarrow$ $AB^2 + BC^2 = AC^2$

30. If the median of the following frequency distribution is 32.5. Find the values of f_1 and f_2.

Classes	**0 – 10**	**10 – 20**	**20 – 30**	**30 – 40**	**40 – 50**	**50 – 60**	**60 – 70**	**Total**
Frequency	f_1	**5**	**9**	**12**	f_2	**3**	**2**	**40**

Or

The marks obtained by 100 students of a class in an examination are given below:

Marks	No. of students
0 – 5	2
5 – 10	5
10 – 15	6
15 – 20	8
20 – 25	10
25 – 30	25
30 – 35	20
35 – 40	18
40 – 45	4
45 – 50	2

Draw 'a less than' type cumulative frequency curves (ogive). Hence find median:

Sol.

Class Interval	Frequency	Cumulative frequency
0 – 10	f_1	f_1
10 – 20	5	$f_1 + 5$
20 – 30	9	$f_1 + 14$
30 – 40	12	$f_1 + 26$
40 – 50	f_2	$f_1 + f_2 + 26$
50 – 60	3	$f_1 + f_2 + 29$
60 – 70	2	$f_1 + f_2 + 31$
Total	$\Sigma f_i = 40$	

$$\text{Median} = 32.5, l = 30, f = 12, c.f. = f_1 + 14$$

$$h = 10, n = 40$$

$$\text{Median} = l + \left[\frac{\frac{n}{2} - cf}{f}\right] \times h$$

$$32.5 = 30 + \left[\frac{\frac{40}{2} - f_1 - 14}{12}\right] \times 10$$

$$32.5 = 30 + \left[\frac{20 - f_1 - 14}{12}\right] \times 10$$

$$32.5 = 30 + (6 - f_1) \times \frac{5}{6}$$

$$2.5 = (6 - f_1) \times \frac{5}{6}$$

$$15 = 5(6 - f_1)$$

$$3 = 6 - f_1 \Rightarrow f_1 = 3$$

Also,

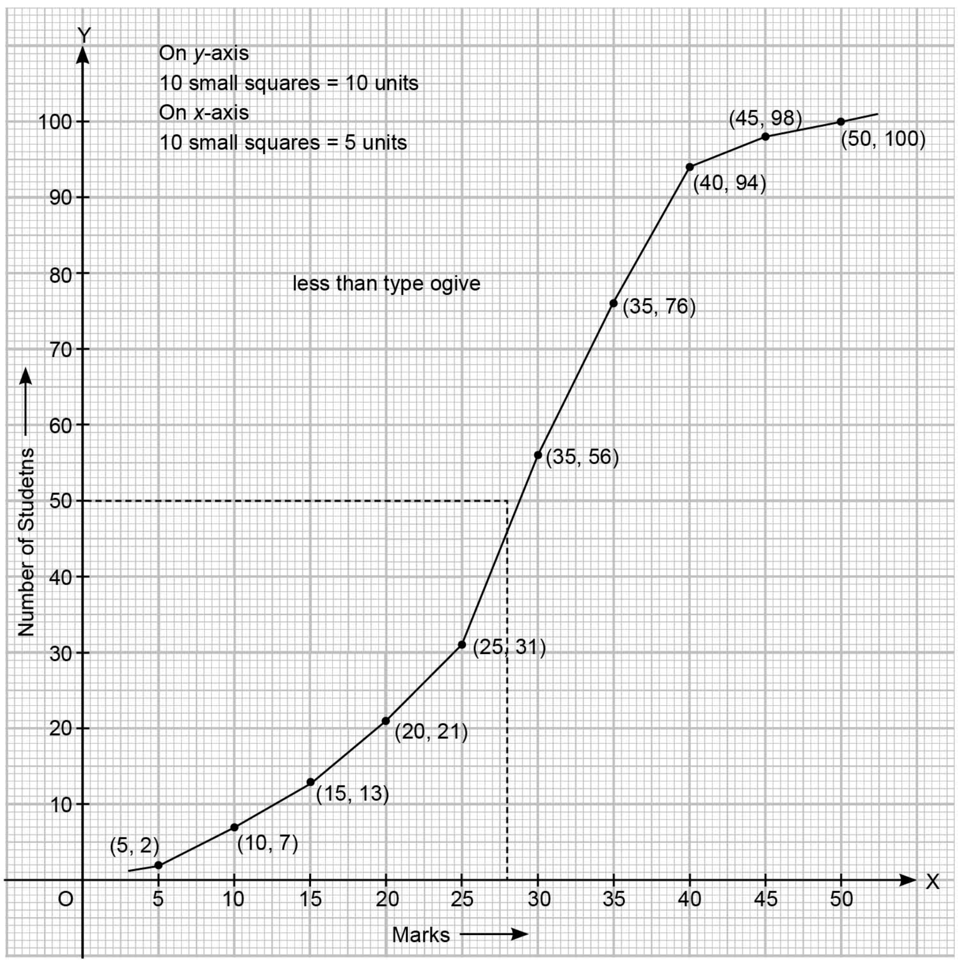
$$f_1 + f_2 + 31 = 40$$

$$3 + f_2 + 31 = 40$$

$$f_2 = 40 - 34 \Rightarrow f_2 = 6$$

Or

Marks less than	No. of students
5	2
10	7
15	13
20	21
25	31
30	56
35	76
40	94
45	98
50	100



Class Interval	Frequency	C.F.
0 – 5	2	2
5 – 10	5	7
10 – 15	6	13
15 – 20	8	21
20 – 25	10	31
25 – 30	25	56
30 – 35	20	76
35 – 40	18	94
40 – 45	4	98
45 – 50	2	100
	100	

N = 50

$$\text{Median} = l + \frac{\frac{N}{2} - F}{f}$$

$$= 25 + \frac{50 - 31}{25}$$

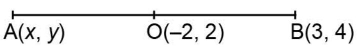
$$= 25.76$$

SET-II UNCOMMON QUESTIONS TO SET-I

1. Find the coordinates of a point A, where AB is a diameter of the circle with centre (–2, 2) and B is the point with coordinates (3, 4).

Sol. AB is diameter of circle.

Centre (–2, 2) is mid-point of diameter AB.



$$\therefore \quad -2 = \frac{x+3}{2} \Rightarrow x + 3 = -4 \Rightarrow x = -7$$

$$2 = \frac{y+4}{2} \Rightarrow y + 4 = 4 \Rightarrow y = 0$$

$\therefore$ coordinates of A are(–7, 0)

2. Find the value of k for which the following pair of linear equations have infinitely many solutions. $2x + 3y = 7$, $(k + 1)x + (2k - 1)y = 4k + 1$

Sol. For infinitely many solutions:

$$\frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$$

$$\Rightarrow \quad \frac{2}{k+1} = \frac{3}{2k-1} = \frac{7}{4k+1}$$

$$\Rightarrow \quad 2(2k-1) = 3(k+1) \text{ and } \frac{3}{2k-1} = \frac{7}{4k+1}$$

$$4k - 2 = 3k + 3 \quad \text{and} \quad 3(4k + 1) = 7(2k - 1)$$

$$k = 5 \qquad 12k + 3 = 14k - 7$$

$$10 = 2k \Rightarrow k = 5$$

3. The arithmetic mean of the following frequency distribution is 53. Find the value of k.

Class	**0 – 20**	**20 – 40**	**40 – 60**	**60 – 80**	**80 – 100**
Frequency	**12**	**15**	**32**	***k***	**13**

Sol.

Class interval	Frequency	mid value x_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$
0 – 20	12	10	–2	–24
20 – 40	15	30	–1	–15
40 – 60	32	50	0	0
60 – 80	k	70	1	k
80 – 100	13	90	2	26
	$\Sigma f_i = 72 + k$			$\Sigma f_i u_i = k - 13$

$$a = 50$$

$$\bar{u} = \frac{\Sigma f_i u_i}{\Sigma f_i} \Rightarrow \bar{u} = \frac{k - 13}{72 + k}$$

$$\bar{x} = a + h\bar{u} \Rightarrow 53 = 50 + 20\left(\frac{k - 13}{72 + k}\right)$$

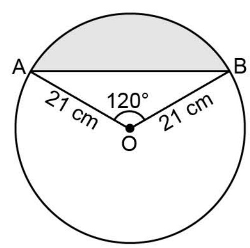
$$3(72 + k) = 20k - 260$$

$$216 + 260 = 17k$$

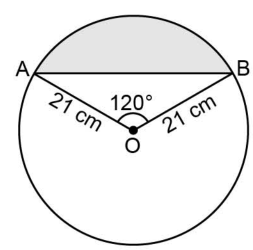
$$k = \frac{476}{17}$$

$$k = 28$$

4. Find the area of the segment shown in Fig., if radius of the circle is 21 cm and ∠AOB = 120° $\left(\textbf{Use } \pi = \frac{22}{7}\right)$



Sol.



$$\text{Radius of circle} = 21 \text{ cm}$$

$$\text{Area of sector AOB} = \frac{\pi(21)^2\, 120°}{360°}$$

$$= \frac{22}{7} \times \frac{21 \times 21}{3} = 462 \text{ cm}^2$$

$$\text{area of } \Delta\text{AOB} = \frac{1}{2} r^2 \cdot \sin\theta$$

$$= \frac{1}{2} \times (21)^2 \times \sin 120°$$

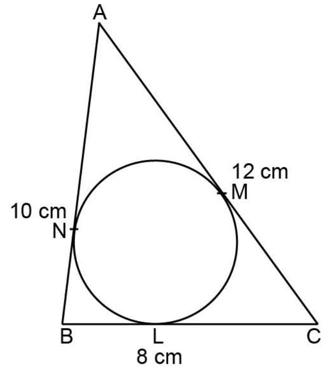
$$= \frac{1}{2} \times 21 \times 21 \times \frac{\sqrt{3}}{2} \qquad \left(\sin 120° = \frac{\sqrt{3}}{2}\right)$$

$$= \frac{441}{4}\sqrt{3}$$

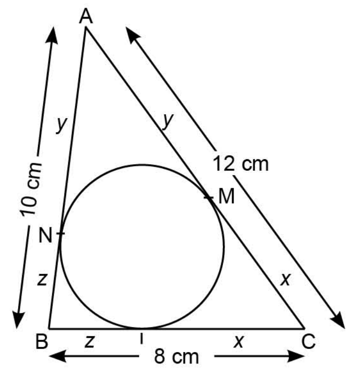
$$\text{area of minor segment} = 462 - \frac{441}{4}\sqrt{3}$$

$$= 462 - 190.95 = 271.05 \text{ cm}^2$$

3. In Fig., a circle is inscribed in ΔABC having sides BC = 8 cm, AB = 10 cm and AC = 12 cm. Find the lengths of BL, CM and AN.



Sol. Let lengths of tangents drawn from common external point are equal.



$\therefore$ $z + x = 8$ ...(*i*)

$x + y = 12$ ...(*ii*)

$z + y = 10$ ...(*iii*)

Adding (*i*), (*ii*) and (*iii*)

$$2(x + y + z) = 30$$

$\Rightarrow$ $x + y + z = 15$ ...(*iv*)

From eq. (*iv*) and eq. (*i*)

$\Rightarrow$ $y + 8 = 15 \Rightarrow y = 7$ cm

From eq (*ii*) $x = 12 - y = 12 - 7 = 5$ cm

From eq (*iii*) $z = 10 - y = 10 - 7 = 3$ cm

$\therefore$ BL = z = 3 cm, AN = y = 7 cm, CM = x = 5 cm.

4. Prove that $\dfrac{\tan^2 A}{\tan^2 A - 1} + \dfrac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A} = \dfrac{1}{1 - 2\cos^2 A}$.

Sol. $\dfrac{\tan^2 A}{\tan^2 A - 1} + \dfrac{\text{cosec}^2 A}{\sec^2 A - \text{cosec}^2 A} = \dfrac{1}{1 - 2\cos^2 A}$

$$\frac{\sin^2 A \cdot \cos^2 A}{\cos^2 A(\sin^2 A - \cos^2 A)} + \frac{\sin^2 A \cos^2 A}{\sin^2 A(\sin^2 A - \cos^2 A)}$$

$$\frac{\sin^2 A}{\sin^2 A - \cos^2 A} + \frac{\cos^2 A}{\sin^2 A - \cos^2 A} = \frac{1}{\sin^2 A - \cos^2 A} = \frac{1}{1 - \cos^2 A - \cos^2 A} = \frac{1}{1 - 2\cos^2 A}$$

5. The first term of an AP is 3, the last term is 83 and the sum of all its terms is 903. Find the number of terms and the common difference of the AP.

Sol. Here $a = 3$ and $a_n = 83$.

Let number of terms be n.

$$S_n = 903$$

$\therefore$ $S_n = \dfrac{n}{2}(a + a_n) \Rightarrow 903 = \dfrac{n}{2}[3 + 83]$

$$1806 = n(86)$$

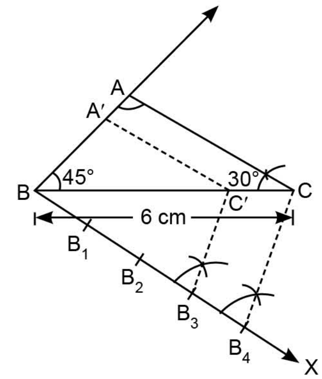
$$n = \frac{1806}{86} = 21$$

$\therefore$ $a_n = a + (n - 1)d$

$$83 = 3 + (20)d \Rightarrow 80 = 20d \Rightarrow d = 4$$

8. Construct a triangle ABC with sides BC = 6 cm, ∠B = 45°, ∠A = 105°. Then construct another triangle whose sides are $\frac{3}{4}$ times the corresponding sides of the ΔABC.

Sol. In ΔABC,



$$\angle A + \angle B + \angle C = 180°$$
$$105° + 45° + \angle C = 180°$$
$$150° + \angle C = 180°$$
$$\angle C = 30°$$

Steps of Construction:

(*a*) Draw BC = 6 cm.

(*b*) At B draw ∠B = 45° and ∠C = 30°.

(*c*) Draw acute angle ∠CBX, below BC at B.

(*d*) Mark B_1, B_2, B_3, B_4 on BX such that $BB_1 = B_1B_2 = B_2B_3 = B_3B_4$

(*e*) Join B_4C, draw $B_3C' \parallel B_4C$, where C′ is a point on BC.

(*f*) Draw C′A′ || AC, where A′ is a point on BA.

(*g*) ΔA′BC′ is the required triangle.

SET-III UNCOMMON QUESTIONS TO SET-I & SET-II

1. Two positive integers a and b can be written as $a = x^3y^2$ and $b = xy^3$. x, y are prime numbers, then LCM(a, b).

Sol.
$$a = x^3y^2 \text{ and } b = xy^3$$
$$\text{HCF} = xy^2$$
$$\text{LCM} = \frac{a \times b}{\text{HCF}} = \frac{x^3y^2 \times xy^3}{xy^2} = x^3y^3$$

2. Find, how many two digit natural numbers are divisible by 7.

Or

If the sum of first n terms of an AP is n^2, then find its 10th term.

Sol. Two digit numbers divisible by 7 are 14, 21, ... 98

$a = 14, d = 21 - 14 = 7, a_n = 98$

so, $a_n = a + (n-1)d$ (where n is number of terms)

$$98 = 14 + (n-1)7$$
$$98 = 14 + 7n - 7$$
$$91 = 7n$$
$$n = 13$$

Or

Here, S_n is sum of n terms of an AP.

∴ $S_n = n^2$ (given)

nth term of the AP is given by

$$a_n = S_n - S_{n-1}$$

$$a_n = n^2 - (n-1)^2$$
$$= n^2 - (n^2 + 1 - 2n) = 2n - 1$$
$$a_n = 2n - 1$$
Put $$n = 10$$
$$a_{10} = 2(10) - 1 = 19$$

Here 10th term is 19.

3. Find all zeros of the polynomial $3x^3 + 10x^2 - 9x - 4$ if one of its zero is 1.

Sol. 1 is one of the zero of $P(x) = 3x^3 + 10x^2 - 9x - 4$

$\Rightarrow x = 1$ satisfy the polynomial $P(x)$.

$\therefore$ $(x - 1)$ is a factor of $P(x)$

Hence $(x - 1)$ divides $P(x)$ completely

$$\begin{array}{r|l} x-1 & 3x^3 + 10x^2 - 9x - 4 \;(\; 3x^2 + 13x + 4 \\ & \underline{\underset{-}{3x^3} \;\underset{+}{-\, 3x^2}} \\ & \quad 13x^2 - 9x - 4 \\ & \quad \underline{\underset{-}{13x^2} \;\underset{+}{-\, 13x}} \\ & \qquad 4x - 4 \\ & \qquad \underline{\underset{-}{4x} \;\underset{+}{-\, 4}} \\ & \qquad\quad 0 \end{array}$$

$3x^3 + 10x^2 - 9x - 4 = (x - 1)(3x^2 + 13x + 4)$

For other zeros, put $3x^2 + 13x + 4 = 0$

$$3x^2 + 12x + x + 4 = 0$$
$$3x(x + 4) + 1(x + 4) = 0$$
$$(x + 4)(3x + 1) = 0$$
$$\Rightarrow \quad x = -4 \text{ or } x = -\frac{1}{3}$$

Hence, other zeros are -4 and $-\frac{1}{3}$.

4. Prove that $\frac{2+\sqrt{3}}{5}$ is an irrational number, given that $\sqrt{3}$ is an irrational number.

Sol. It is given that $\sqrt{3}$ is an irrational number.

Let $\frac{2+\sqrt{3}}{5}$ is a rational number.

$\therefore$ $\frac{2+\sqrt{3}}{5}$ can be written as $\frac{a}{b}$, where a and b are coprime and $b \neq 0$.

$$\frac{2+\sqrt{3}}{5} = \frac{a}{b}$$
$$\sqrt{3} = \frac{5a}{b} - 2$$

Here, RHS represents a rational number but LHS represents $\sqrt{3}$, which is an irrational number (given). This gives a contradiction.

$\therefore$ Our assumption is wrong.

$\therefore\ \dfrac{2+\sqrt{3}}{5}$ is an irrational number.

4. If sec θ = $x + \dfrac{1}{4x}, x \neq 0$, find (sec θ + tan θ).

Sol.
$$\sec\theta = x + \frac{1}{4x} \quad \text{...(i)}$$

We know that

$$\sec^2\theta = 1 + \tan^2\theta$$

$$\Rightarrow \quad 1 + \tan^2\theta = \left(x + \frac{1}{4x}\right)^2$$

$$1 + \tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2}$$

$$\tan^2\theta = x^2 + \frac{1}{16x^2} + \frac{1}{2} - 1$$

$$= x^2 + \frac{1}{16x^2} - \frac{1}{2}$$

$$\tan^2\theta = \left(x - \frac{1}{4x}\right)^2$$

This gives

$$\tan\theta = \pm\left(x - \frac{1}{4x}\right) \quad \text{...(ii)}$$

On adding (*i*) and (*ii*)

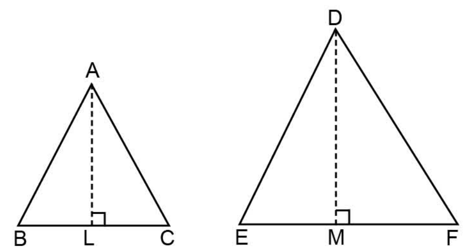
$$\sec\theta + \tan\theta = x + \frac{1}{4x} + x - \frac{1}{4x} = 2x$$

or
$$\sec\theta + \tan\theta = x + \frac{1}{4x} - x + \frac{1}{4x} = \frac{1}{2x}$$

5. Prove that the ratio of the areas of two similar triangles is equal to the square of the ratio of their corresponding sides.

Sol. Given: Two triangles ABC and DEF such that ΔABC ~ ΔDEF.

To prove: $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{AB^2}{DE^2} = \dfrac{BC^2}{EF^2} = \dfrac{AC^2}{DF^2}$



Construction: Draw AL ⊥ BC and DM ⊥ EF

Proof: In ΔALB and ΔDME,

$\angle ALB = \angle DME = 90°$ [By Construction]

$\angle B = \angle E$ [∵ ΔABC ~ ΔDEF]

∴ By AA criterion of similarity,

ΔALB ~ ΔDME

So, $\dfrac{AL}{DM} = \dfrac{AB}{DE}$ ...(*i*)

$\dfrac{AB}{DE} = \dfrac{BC}{EF} = \dfrac{AC}{DF}$ ...(*ii*) [Ratio of corresponding sides of similar triangles are equal]

From (*i*) and (*ii*), we get

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF} = \frac{AL}{DM} \quad \text{...(iii)}$$

Now, $\dfrac{ar(\Delta ABC)}{ar(\Delta DEF)} = \dfrac{\frac{1}{2}(BC \times AL)}{\frac{1}{2}(EF \times DM)}$

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC}{EF} \times \frac{AL}{DM}$$

$$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{BC \times BC}{EF \times EF} = \frac{BC^2}{EF^2}$$ [From (*iii*)]

And $$\frac{BC^2}{EF^2} = \frac{AB^2}{DE^2} = \frac{AC^2}{DF^2}$$ [From (*iii*)]

Hence, $$\frac{ar(\Delta ABC)}{ar(\Delta DEF)} = \frac{AB^2}{DE^2} = \frac{BC^2}{EF^2} = \frac{AC^2}{DF^2}$$ Hence proved.

5. The following distribution gives the daily income of 50 workers of a factory.

Daily income (in ₹)	200-220	220-240	240-260	260-280	280-300
Number of workers	12	14	8	6	10

Convert the distribution above to a 'less than type' cumulative frequency distribution and draw its ogive.

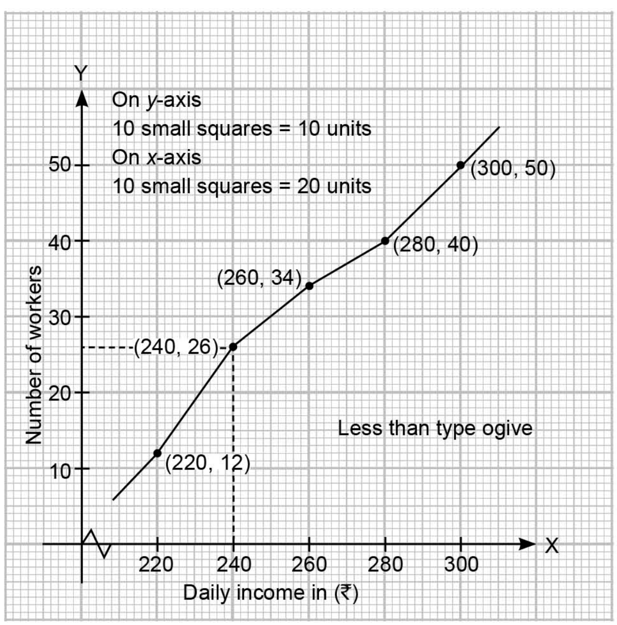
Or

The table below shows the daily expenditure on food of 25 households in a locality. Find the mean daily expenditure on food.

Daily expenditure (in ₹):	100-150	150-200	200-250	250-300	300-350
Number of households:	4	5	12	2	2

Sol.

Less than daily income in (₹)	Frequency
220	12
240	26
260	34
280	40
300	50



Or

Daily Income in (₹)	**mid-value**	$u_i = \frac{x_i - a}{h}$	f_i	$f_i u_i$
100 – 150	125	–2	4	–8
150 – 200	175	–1	5	–5
200 – 250	225	0	12	0
250 – 300	275	1	2	2
300 – 350	325	2	2	4
			25	–7

$$\text{Mean} = a + \left[\frac{\Sigma f_i u_i}{\Sigma f_i}\right] h$$

$$a = 225$$

$$h = 50$$

$$\text{Mean} = 225 + \left[\frac{-7}{25}\right] \times 50$$

$$= 225 - 14 = 211$$