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# OBJECTIVE TYPE QUESTIONS

# (MATHEMATICS)


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CLASS
**12**
TERM 2


RACHNA
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# Part-I

**[Multiple Choice Questions, Assertion-Reason Questions and Case-based Questions]**

# 7 INTEGRALS

## Multiple Choice Questions

**1.** If $\frac{d}{dx}f(x) = g(x)$, then antiderivative of $g(x)$ is

(*a*) $f(x)$ (*b*) $g(x)$ (*c*) $\frac{1}{2}[f(x)]^2$ (*d*) $\frac{1}{2}[g(x)]^2$

**Sol.** (*a*), as $\frac{d}{dx}f(x) = g(x) \Rightarrow \int g(x)\,dx = f(x)$.

**2.** Given $\int 2^x dx = f(x) + C$, then $f(x)$ is

(*a*) $2^x$ (*b*) $2^x \log_e 2$ (*c*) $\frac{2^x}{\log_e 2}$ (*d*) $\frac{2^{x+1}}{x+1}$

**Sol.** (*c*), as $\frac{d}{dx}\left(\frac{2^x}{\log_e 2}\right) = \frac{1}{\log_e 2}\cdot 2^x \cdot \log_e 2 = 2^x$.

**3.** $\int \frac{1}{\sin^2 x \cos^2 x}dx$ is equal to

(*a*) $\sin^2 x - \cos^2 x + C$ (*b*) $-1$

(*c*) $\tan x + \cot x + C$ (*d*) $\tan x - \cot x + C$

**Sol.** (*d*), as $\int \frac{1}{\sin^2 x \cos^2 x}dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x}dx = \int (\sec^2 x + \text{cosec}^2 x)\,dx = \tan x - \cot x + C$

**4.** $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta}dx$ is equal to **[KVS]**

(*a*) $2(\sin x + x\cos\theta) + C$ (*b*) $2(\sin x - x\cos\theta) + C$

(*c*) $2(\sin x + 2x\cos\theta) + C$ (*d*) $2(\sin x - 2x\cos\theta) + C$

**Sol.** (*a*), as $\int \frac{2(\cos^2 x - \cos^2\theta)}{\cos x - \cos\theta}dx$ (using $\cos 2x = 2\cos^2 x - 1$)

$= 2\int (\cos x + \cos\theta)\,dx = 2\sin x + 2x\cdot\cos\theta + C$

**5.** $\int \cot^2 x \, dx$ equals to

($a$) $\cot x - x + C$ ($b$) $\cot x + x + C$

($c$) $-\cot x + x + C$ ($d$) $-\cot x - x + C$

**Sol.** ($d$), $\int (\text{cosec}^2 x - 1) dx = -\cot x - x + C$

**6.** $\int \frac{\sin x + \cos x}{\sqrt{1 + \sin 2x}} dx$, $\frac{3\pi}{4} < x < \frac{7\pi}{4}$ is equal to

($a$) $\log|\sin x + \cos x|$ ($b$) $x$ ($c$) $\log|x|$ ($d$) $-x$

**Sol.** ($d$), as $\int \frac{\sin x + \cos x}{|\sin x + \cos x|} dx \Rightarrow -\int 1 \cdot dx = -x + C$ {as $\sin x + \cos x < 0$ for $\frac{3\pi}{4} < x < \frac{7\pi}{4}$}

**7.** $\int \frac{3x^2 + 3^x \cdot \log 3}{3^x + x^3} dx$ is equal to

($a$) $3^x + x^3 + C$ ($b$) $\log|3^x + x^3| + C$

($c$) $3x^2 + 3^x \log_e 3 + C$ ($d$) $\log|3x^2 + 3^x \log_e 3| + C$

**Sol.** ($b$), as $\int \frac{1}{t} dt = \log|t| + C$ | Let $3^x + x^3 = t \Rightarrow (3^x \cdot \log_e 3 + 3x^2)dx = dt$

$= \log|3^x + x^3| + C$

**8.** If $\int \sec^2(7 - 4x) dx = a \tan(7 - 4x) + C$, then value of $a$ is

($a$) 7 ($b$) $-4$ ($c$) 3 ($d$) $-\frac{1}{4}$

**Sol.** ($d$), $\int \sec^2(7 - 4x) dx = \frac{\tan(7 - 4x)}{-4} + C = -\frac{1}{4}\tan(7 - 4x) + C.$

**9.** $\int \frac{x + \cos 6x}{3x^2 + \sin 6x} dx$ is equal to

($a$) $3x^2 + \sin 6x + C$ ($b$) $1 - 6\sin 6x + C$

($c$) $\log|3x^2 + \sin 6x| + C$ ($d$) $\frac{1}{6}\log|3x^2 + \sin 6x| + C$

**Sol.** ($d$), as $\int \frac{x + \cos 6x}{3x^2 + \sin 6x} dx = \frac{1}{6}\int \frac{1}{t} dt = \frac{1}{6}\log|t| + C$

$= \frac{1}{6}\log|3x^2 + \sin 6x| + C$

Let $3x^2 + \sin 6x = t$

$\Rightarrow (6x + 6\cos 6x)dx = dt$

$\Rightarrow (x + \cos 6x)dx = \frac{1}{6} dt$

**10.** $\int \sec^2(7 - x) dx$ is equal to

($a$) $\tan(7 - x) + C$ ($b$) $2\sec^2(7 - x)\tan x + C$

($c$) $\sec^3(7 - x) + C$ ($d$) $-\tan(7 - x) + C$

**Sol.** ($d$), as $\int \sec^2(7 - x) dx = -\int \sec^2 t \, dt$

$= -\tan t + C = -\tan(7 - x) + C$

Let $7 - x = t$,

$\Rightarrow -dx = dt$

**11.** $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx$ is equal to

(*a*) $\cos\sqrt{x} + C$ (*b*) $2\cos\sqrt{x} + C$ (*c*) $-2\cos\sqrt{x} + C$ (*d*) $\sqrt{x}\cos\sqrt{x} + C$

**Sol.** (*c*), as $\int \frac{\sin\sqrt{x}}{\sqrt{x}} dx = 2\int \sin t \, dt$

$= -2\cos t$

$= -2\cos\sqrt{x} + C$

Let $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}} dx = dt$

$\Rightarrow \frac{1}{\sqrt{x}} dx = 2dt$

**12.** $\int 2x \sin(x^2+1)\, dx$ is equal to

(*a*) $\cos(x^2 + 1) + C$ (*b*) $2\sin(x^2 + 1) + C$

(*c*) $\sin(x^2 + 1) + C$ (*d*) $-\cos(x^2 + 1) + C$

**Sol.** (*d*), as $\int 2x\sin(x^2+1)\, dx = \int \sin t \, dt$

$= -\cos t + C = -\cos(x^2 + 1) + C$

Let $x^2 + 1 = t$

$\Rightarrow 2x dx = dt$

**13.** $\int \frac{x^2}{1+x^3} dx$ is equal to

(*a*) $\frac{2}{3x} + C$ (*b*) $2\log x + C$

(*c*) $\frac{1}{3}\log|1+x^3| + C$ (*d*) $3\log(1 + x^3) + C$

**Sol.** (*c*), as $\int \frac{x^2}{1+x^3} dx = \frac{1}{3}\int \frac{1}{t} dt = \frac{1}{3}\log|t| + C$

$= \frac{1}{3}\log|1 + x^3| + C$

Let $1 + x^3 = t$

$\Rightarrow 3x^2 dx = dt$

$\Rightarrow x^2 dx = \frac{1}{3} dt$

**14.** $\int \frac{x^2+4x}{x^3+6x^2+5} dx$ is equal to

(*a*) $|x^3 + 6x^2 + 5| + C$ (*b*) $\frac{1}{3}\log|x^3 + 6x^2 + 5| + C$

(*c*) $\log|x^2 + 6x| + C$ (*d*) $\frac{1}{2}\log|x^2 + 4x| + C$

**Sol.** (*b*), as $\int \frac{x^2+4x}{x^3+6x^2+5} dx = \frac{1}{3}\int \frac{1}{t} dt$

$= \frac{1}{3}\log|t| + C = \frac{1}{3}\log|x^3 + 6x^2 + 5| + C$

Let $x^3 + 6x^2 + 5 = t$

$\Rightarrow (3x^2 + 12x) dx = dt$

$\Rightarrow (x^2 + 4x) dx = \frac{1}{3} dt$

**15.** $\int \frac{\sin^6 x}{\cos^8 x} dx$ is equal to

(*a*) $\frac{\sin^7 x}{\cos^9 x} + C$ (*b*) $\frac{1}{7}\tan^7 x + C$ (*c*) $\tan^6 x + C$ (*d*) $\sec^8 x + C$

**Sol.** (*b*), as $\int \tan^6 x \cdot \sec^2 x \, dx = \int t^6 \, dt = \frac{t^7}{7} + C = \frac{\tan^7 x}{7} + C$

Let $\tan x = t \Rightarrow \sec^2 x \, dx = dt$

**16.** $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$ equals to

(*a*) $2e^{\sqrt{x}} + C$ (*b*) $\frac{1}{\sqrt{x}} + C$ (*c*) $2\sqrt{x} + C$ (*d*) $e^{\sqrt{x}} + C$

**Sol.** (*a*), $2\int \cdot e^t\,dt = 2e^t + C = 2e^{\sqrt{x}} + C$ |Let $\sqrt{x} = t \Rightarrow \frac{1}{2\sqrt{x}}dx = dt$

**17.** The value of $\lambda$ for which $\int \frac{4x^3 + \lambda 4^x}{4^x + x^4}dx = \log|4^x + x^4|$ is

(*a*) 1 (*b*) $\log_e 4$ (*c*) $\text{loe}_4\, e$ (*d*) 4

**Sol.** (*b*), as $\frac{d}{dx}\log|4^x + x^4| = \frac{1}{4^x + x^4}\cdot(4^x \cdot \log_e 4 + 4x^3) = \frac{4x^3 + \log_e 4 \cdot 4^x}{4^x + x^4}$

$\Rightarrow \quad \lambda = \log_e 4$

**18.** If $\int \frac{1}{\sqrt{4 - 9x^2}}dx = \frac{1}{3}\sin^{-1}(ax) + C$, then value of $a$ is

(*a*) 2 (*b*) 4 (*c*) $\frac{3}{2}$ (*d*) $\frac{2}{3}$

**Sol.** (*c*), as $\int \frac{1}{\sqrt{4 - 9x^2}}dx = \frac{1}{3}\int \frac{1}{\sqrt{\left(\frac{2}{3}\right)^2 - x^2}}dx$

$$= \frac{1}{3}\sin^{-1}\left(\frac{3x}{2}\right) + C$$

$$\Rightarrow \quad a = \frac{3}{2}.$$

**19.** If $\int \frac{1}{(3 + x)^2 + 1}dx = a\tan^{-1}(3 + x) + C$, then value of $a$ is

(*a*) $\frac{1}{3}$ (*b*) 3 (*c*) 1 (*d*) –1

**Sol.** (*c*), as $\int \frac{1}{(3 + x)^2 + 1}dx = \frac{1}{1}\tan^{-1}\frac{3 + x}{1} = \tan^{-1}(3 + x) + C$

**20.** $\int \frac{1}{\sqrt{4 - x^2}}dx$ is equal to

(*a*) $\log|x + \sqrt{4 - x^2}| + C$ (*b*) $\sin^{-1}\frac{2}{x} + C$

(*c*) $\sin^{-1}\frac{x}{2} + C$ (*d*) $\frac{1}{\log|x + \sqrt{4 - x^2}|} + C$

**Sol.** (*c*), as $\int \frac{1}{\sqrt{4 - x^2}}dx = \sin^{-1}\frac{x}{2} + C$

**21.** If $x = \int_0^y \frac{dt}{\sqrt{1+9t^2}}$ and $\frac{d^2y}{dx^2} = ay$, then value of $a$ is equal to

(*a*) 3 (*b*) 6 (*c*) 9 (*d*) 1

**Sol.** (*c*), as $\frac{dx}{dy} = \frac{1}{\sqrt{1+9y^2}} \Rightarrow \frac{dy}{dx} = \sqrt{1+9y^2} \Rightarrow \frac{d^2y}{dx^2} = \frac{18y}{2\sqrt{1+9y^2}} \cdot \frac{dy}{dx}$

$$\Rightarrow \frac{d^2y}{dx^2} = 9y \cdot \frac{1}{\sqrt{1+9y^2}} \cdot \sqrt{1+9y^2} = 9y$$

Hence, $a = 9$

**22.** $\int e^x \left(\frac{1-x}{1+x^2}\right)^2 dx$ is equal to

(*a*) $\frac{e^x}{1+x^2}+C$ (*b*) $-\frac{e^x}{1+x^2}+C$ (*c*) $\frac{e^x}{(1+x^2)^2}+C$ (*d*) $-\frac{e^x}{(1+x^2)^2}+C$

**Sol.** (*a*), as $\int e^x \left(\frac{1+x^2-2x}{(1+x^2)^2}\right)dx = \int e^x \left\{\frac{1}{1+x^2} - \frac{2x}{(1+x^2)^2}\right\}dx$

$$f(x) = \frac{1}{1+x^2}; f'(x) = \frac{-2x}{(1+x^2)^2}$$

Using $\int e^x \{f(x) + f'(x)\}dx = e^x \cdot f(x) + C = e^x \cdot \frac{1}{1+x^2} + C$

**23.** $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x}$ equals to

(*a*) 0 (*b*) $\frac{1}{2}$ (*c*) 1 (*d*) $\frac{3}{2}$

**Sol.** (*c*), as $\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)} = \int_0^{\frac{\pi}{2}} \frac{1}{2}\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx$

$$= \frac{1}{2} \cdot \left[\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{-\frac{1}{2}}\right]_0^{\frac{\pi}{2}} = -\tan\left(\frac{\pi}{4}-\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{4}-0\right) = 1.$$

**24.** Let $I_1 = \int_1^2 \frac{dx}{\sqrt{1+x^2}}$ and $I_2 = \int_1^2 \frac{dx}{x}$, then

(*a*) $I_1 > I_2$ (*b*) $I_2 > I_1$ (*c*) $I_1 = I_2$ (*d*) $I_1 > 2I_2$

**Sol.** (*b*), we have $1 + x^2 > x^2 \Rightarrow \sqrt{1+x^2} > x$ for $x \in [1, 2]$

$$\Rightarrow \frac{1}{\sqrt{1+x^2}} < \frac{1}{x} \Rightarrow \int_1^2 \frac{1}{\sqrt{1+x^2}}dx < \int_1^2 \frac{dx}{x} \Rightarrow I_1 < I_2$$

**25.** If $a$ is such that $\int_0^a x\,dx \le a + 4$, then

(a) $0 \le a \le 4$ (b) $-2 \le a \le 0$

(c) $a \le -2$ or $a \le 4$ (d) $-2 \le a \le 4$

**Sol.** (d), as $\int_0^a x\,dx \le a + 4 \Rightarrow \frac{a^2}{2} \le a + 4$

$\Rightarrow$ $a^2 - 2a - 8 \le 0$

$\Rightarrow$ $(a-1)^2 \le (3)^2 \Rightarrow -3 \le a - 1 \le 3$

$\Rightarrow$ $-2 \le a \le 4$

**26.** If $\int_0^a \frac{1}{1+4x^2}\,dx = \frac{\pi}{8}$, then value of $a$ is

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) 2 (d) 1

**Sol.** (b), as $\int_0^a \frac{1}{1+(2x)^2}\,dx = \left[\frac{1}{2}\tan^{-1}(2x)\right]_0^a = \frac{\pi}{8} \Rightarrow \frac{1}{2}\tan^{-1}(2a) = \frac{\pi}{8}$

$\Rightarrow$ $2a = \tan\frac{\pi}{4} \Rightarrow a = \frac{1}{2}.$

**27.** $\int_{-1}^{1} |(1-x)|\,dx$ is equal to

(a) $\frac{1}{2}$ (b) $-1$ (c) 2 (d) 1

**Sol.** (c), as $\int_{-1}^{1} |1-x|\,dx = \int_{-1}^{1} (1-x)\,dx$, $1 - x \ge 0$ for $-1 \le x \le 1$

$$= \left[x - \frac{x^2}{2}\right]_{-1}^{1} = \left(1 - \frac{1}{2}\right) - \left(-1 - \frac{1}{2}\right) = \frac{1}{2} + \frac{3}{2} = 2$$

**28.** The value of $\int_{-\pi}^{\pi} \sin^3 x \cos^2 x\,dx$ is

(a) 0 (b) $\frac{1}{2}$ (c) 2 (d) 1

**Sol.** (a), as $f(x) = \sin^3 x \cdot \cos^2 x\,dx$ is an odd function.

**29.** $\int \frac{1}{x^2(x^4+1)^{3/4}}\,dx$ is equal to

(a) $-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}} + C$ (b) $(x^4+1)^{\frac{1}{4}} + C$ (c) $\left(1-\frac{1}{x^4}\right)^{\frac{1}{4}} + C$ (d) $-\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}} + C$

**Sol.** (a), as $\int \frac{1}{x^2 \cdot x^3\left(1+\frac{1}{x^4}\right)^{3/4}}\,dx = \int \frac{1}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}\,dx$ we substitute $1 + \frac{1}{x^4} = t$

**30.** $\int \frac{xe^x}{(1+x)^2}\, dx$ is equal to

(*a*) $\frac{e^x}{x+1}+C$ (*b*) $e^x(x+1)+C$ (*c*) $-\frac{e^x}{(x+1)^2}+C$ (*d*) $\frac{e^x}{1+x^2}+C$

**Sol.** (*a*), proceed as $\int e^x\left\{\frac{1}{1+x}-\frac{1}{(1+x)^2}\right\}dx$ and $\int e^x\{f(x)+f'(x)\}dx = e^x f(x)+C$

**31.** If $\int \frac{2^x}{\sqrt{1-4^x}}\, dx = p.\sin^{-1}(2^x)+C$, then '$p$' is equal to

(*a*) $\log_e 2$ (*b*) $\frac{1}{2}\log_e 2$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{\log_e 2}$

**Sol.** (*d*), let $2^x = t \Rightarrow 2^x\cdot \log_e 2\, dx = dt$

$$\therefore \int \frac{2^x}{\sqrt{1-4^x}}dx = \frac{1}{\log_e 2}\cdot\int\frac{1}{\sqrt{1-t^2}}dt = \frac{1}{\log_e 2}\cdot\sin^{-1}(2^x)+C$$

$$\Rightarrow p = \frac{1}{\log_e 2}$$

**32.** The value of integral $\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}dx$ is

(*a*) $\log 2$ (*b*) $\frac{1}{20}\log 2$ (*c*) $\frac{1}{20}\log 3$ (*d*) $\log 5$

**Sol.** (*c*), as
$$\int_0^{\frac{\pi}{4}} \frac{\sin x+\cos x}{9+16\sin 2x}dx = \int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{25-16(1-\sin 2x)}dx$$

$$= \int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{25-16(\sin^2 x+\cos^2 x-2\sin x\cos x)}$$

$$\int_0^{\frac{\pi}{4}}\frac{\sin x+\cos x}{25-16(\cos x-\sin x)^2}dx,$$ put $\cos x-\sin x = t$

**33.** The value of integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x.\log\left(\frac{1+x}{1-x}\right)dx$ is

(*a*) 0 (*b*) $\frac{1}{2}$ (*c*) $\frac{3}{2}$ (*d*) none of these

**Sol.** (*a*), as function is odd function.

**34.** $\int e^{\sqrt{x}}dx$ is equal to

(*a*) $e^{\sqrt{x}}+C$ (*b*) $2\sqrt{x}\,e^{\sqrt{x}}+C$ (*c*) $\frac{e^{\sqrt{x}}}{2\sqrt{x}}+C$ (*d*) $2(\sqrt{x}-1)e^{\sqrt{x}}+C$

**Sol.** (*d*), as
$$\int e^{\sqrt{x}}dx = 2\int \underset{①}{t}\,\underset{②}{e^t}dt$$

Let $x = t^2$
$\Rightarrow dx = 2t\,dt$

$$= 2\left(te^t-\int 1.e^t dt\right) = 2(te^t-e^t)+C = 2\left(\sqrt{x}\,e^{\sqrt{x}}-e^{\sqrt{x}}\right)+C$$

**35.** $\int \frac{1}{\sin^2 x \cos^2 x} dx$ is equal to

(*a*) $\cos^2 x - \sin^2 x + C$ (*b*) $\tan x - \cot x + C$

(*c*) $\sec x + \cos x + C$ (*d*) $\frac{1}{\sin x \cos x} + C$

**Sol.** (*b*), as $\int \frac{1}{\sin^2 x \cos^2 x} dx = \int \frac{\sin^2 x + \cos^2 x}{\sin^2 x \cos^2 x} dx$

$$= \int \left( \frac{1}{\cos^2 x} + \frac{1}{\sin^2 x} \right) dx$$

$$= \int (\sec^2 x + \text{cosec}^2 x)\, dx = \tan x - \cot x + C$$

**36.** $\int \cos^3 x . e^{\log(\sin x)} dx$ is equal to

(*a*) $-\frac{\cos^4 x}{4} + C$ (*b*) $-\frac{\sin^4 x}{4} + C$ (*c*) $\frac{e^{\sin x}}{4} + C$ (*d*) none of these

**Ans.** (*a*)

**37.** $\int \frac{\sqrt{\tan x}}{\sin x . \cos x} dx$ is equal to

(*a*) $2\sqrt{\cot x} + C$ (*b*) $\frac{\sqrt{\tan x}}{2} + C$ (*c*) $2\sqrt{\tan x} + C$ (*d*) none of these

**Ans.** (*c*)

**38.** The value of $\int_0^{\frac{\pi}{2}} \frac{1}{1 + \tan^3 x} dx$ is

(*a*) 0 (*b*) 1 (*c*) $\frac{\pi}{4}$ (*d*) $\frac{\pi}{2}$

**Ans.** (*c*)

**39.** The value of $\int_8^{13} \frac{\sqrt{21 - x}}{\sqrt{x} + \sqrt{21 - x}} dx$ is

(*a*) $\frac{21}{2}$ (*b*) 0 (*c*) $\frac{5}{2}$ (*d*) none of these

**Ans.** (*c*)

**40.** The value of $\int_0^2 x[x]\, dx$ is

(*a*) $\frac{7}{2}$ (*b*) $\frac{3}{2}$ (*c*) $\frac{5}{2}$ (*d*) none of these

**Ans.** (*b*)

## Assertion-Reason Questions

**Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:**

(*a*) If both A and R are true and R is the correct explanation of A.

(*b*) If both A and R are true but R is not the correct explanation of A.

(*c*) A is true R is false.

(*d*) A is false R is true.

**41. Assertion (A):** $\int_0^{10} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}dx = 10$

**Reason (R):** $\int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

**Ans.** (*d*)

Let $$I = \int_0^{10} \frac{\sqrt{x}}{\sqrt{x}+\sqrt{10-x}}dx \quad \text{...}(i)$$

$$I = \int_0^{10} \frac{\sqrt{10-x}}{\sqrt{10-x}+\sqrt{x}}dx \quad \text{...}(ii)$$

On adding (*i*) and (*ii*), we get

$$2I = \int_0^{10} 1\,dx$$

$$2I = [x]_0^{10}$$

$$I = 5$$

A is false, but R is true

**42. Assertion (A):** $\int \frac{e^{\tan^{-1}x}}{1+x^2}dx = e^{\tan^{-1}x} + C$

**Reason (R):** $\frac{d}{dx}e^{\tan^{-1}x} = \frac{e^{\tan^{-1}x}}{1+x^2}$

**Ans.** (*a*)

Let $$I = \int \frac{e^{\tan^{-1}x}}{1+x^2}dx$$

Let $e^{\tan^{-1}x} = t$

$\Rightarrow e^{\tan^{-1}x} \cdot \frac{1}{1+x^2}dx = dt$

So, $$I = \int dt = t + C = e^{\tan^{-1}x} + C$$

A is true, R is also true and the correct explanation of A.

**43. Assertion (A):** $\int e^x \cdot \frac{(x-1)}{(1+x)^3}dx = \frac{e^x}{(x+1)^2} + C$

**Reason (R):** $\int e^x\left(f(x)+f'(x)\right)dx = e^x f(x) + C$

**Ans.** (*a*)

$$I = \int e^x\left[\frac{x-1+1-1}{(x+1)^3}\right]dx = \int e^x\left[\frac{(x+1-2)}{(x+1)^3}\right]dx$$

$$= \int e^x\left(\frac{1}{(x+1)^2} - \frac{2}{(x+1)^3}\right)dx = \frac{e^x}{(x+1)^2} + C$$

$$\because \quad \frac{d}{dx}\left(\frac{1}{(1+x)^2}\right) = \frac{-2}{(x+1)^3}$$

$$\int e^x(f(x)+f'(x))dx = e^x f(x) + C$$

A is true, R is also true and the correct explanation of A.

**44. Assertion (A):** $\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx = \tan(e^x x) + C$

**Reason (R):** $\int f'(x)dx = f(x) + C$

**Ans.** (*b*)

$$\int \frac{e^x(1+x)}{\cos^2(e^x x)}dx = \int \frac{dt}{\cos^2 t} = \int \sec^2 t\, dt = \tan t + C$$

$$= \tan(e^x x) + C \Rightarrow \text{A is true}$$

Let $e^x x = t$

$\Rightarrow$ $(e^x x + 1\cdot e^x)dx = dt$

$\Rightarrow$ $e^x(1 + x)dx = dt$

R is also true, but not the correct explanation of R.

**45. Assertion (A):** $\int_{-\pi}^{\pi} \sin^5 x dx = 0$

**Reason (R):** $\int_a^b f(x^2)dx = \int_a^b f(t)dt$

**Ans.** (*c*)

$$\int_{-\pi}^{\pi} \sin^5 x dx = 0$$

as $$\int_{-a}^{a} f(x)dx = 0$$

if $$f(-x) = -f(x)$$

$$\sin^5(-x) = -\sin^5(x)$$

A is true

$$\int_a^b f(x^2)dx$$

Let $x^2 = t$

$\Rightarrow$ $2xdx = dt$

$\Rightarrow$ $dx = \frac{dt}{2\sqrt{t}}$

$$\int_a^b \frac{f(t)}{2\sqrt{t}}dt \neq \int_a^b f(t)dt$$

R is false

## Case-based Questions

**46.** Three students in a group, studying the concept of the partial fraction, but they were confused while solving the question they didn't have any idea to how to start the solution. One of the student tell them, for the integration by partial fraction, first we must check we are dealing with polynomial and degree of numerator is less than the degree of denominator and proceed for partial fraction, if not, divide numerator by denominator and write it as

$$\frac{\text{Numerator}}{\text{Denominator}} = \text{Quotient} + \frac{\text{Remainder}}{\text{Denominator}}$$

and proceed for partial fraction of $\frac{\text{Remainder}}{\text{Denominator}}$

Based on the above information, answer the following:

(*i*) If the function is $f(x) = \frac{2}{(1-x)(1+x^2)}$, the partial fraction of the given function in constant term $A$, $B$ and $C$ can be written as,

(*a*) $\frac{Ax+B}{1-x} + \frac{C}{1+x^2}$ (*b*) $\frac{Ax}{1-x} + \frac{B+C}{1+x^2}$

(*c*) $\frac{A}{1-x} + \frac{Bx+C}{1+x^2}$ (*d*) $\frac{A}{1+x^2} + \frac{Cx+B}{1+x}$

(*ii*) The value of $A$ is

(*a*) 1 (*b*) 2 (*c*) 0 (*d*) 3

(*iii*) The value of $B$ is

(*a*) 1 (*b*) 3 (*c*) 0 (*d*) 2

(*iv*) The value of $C$ is

(*a*) 2 (*b*) 0 (*c*) 1 (*d*) 3

(*v*) The integration of the function $f(x)$ is

(*a*) $-\log|1-x| + \frac{1}{2}\log|1-x^2| + \tan^{-1}x + C$

(*b*) $\log|1-x| + \frac{1}{2}\log|1+x^2| + \tan^{-1}x + C$

(*c*) $-\log|1-x| + \frac{1}{2}\log|1+x^2| + \tan^{-1}x + C$

(*d*) $-\log|1+x^2| + \frac{1}{2}\log|1-x| + \tan^{-1}x + C$

**Sol.** (*i*) (*c*), $f(x) = \frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$

(*ii*) (*a*), $\frac{2}{(1-x)(1+x^2)} = \frac{A}{1-x} + \frac{Bx+C}{1+x^2}$

$\Rightarrow \quad 2 = A(1+x^2) + (1-x)(Bx+C)$

$\Rightarrow \quad 2 = A + Ax^2 + Bx + C - Bx^2 - Cx$

$\Rightarrow \quad 2 = (A - B)x^2 + (B - C)x + (A + C)$

Comparing coefficient, we get

$A - B = 0, \quad B - C = 0, \quad A + C = 2$

On solving, we get $A = B = C = 1$

(*iii*) (*a*), $B = 1$

(*iv*) (*c*), $C = 1$

(*v*) (*c*), $\int f(x)\,dx = \int \frac{2}{(1-x)(1+x^2)}dx = \int \frac{1}{1-x}dx + \int \frac{x+1}{1+x^2}dx$

$= -\log|1 - x| + \int \frac{x}{1+x^2}dx + \int \frac{1}{1+x^2}dx$

$= -\log|1 - x| + \frac{1}{2}\log|1 + x^2| + \tan^{-1}x + C$

**47.** The definite integrals are used to calculate the area bounded by the simple curve of the given function with in the given limit. The definite integral is given by

$$\int_a^b f(x)\,dx = g(b) - g(a)$$

Based on the above information, answer the following:

(*i*) If the definite integral is given by $\int_a^b f(x)\,dx$ then it can be written as

(*a*) $\int_b^a f(x)\,dx$ (*b*) $\int_a^b f(a+b-x)\,dx$

(*c*) $\int_0^a f(x)\,dx$ (*d*) $\int_{-a}^a f(x)\,dx$

(*ii*) If $a = b$, then the value of the definite integral will be

(*a*) $2\int_0^a f(x)\,dx$ (*b*) $-\int_a^b f(x)\,dx$ (*c*) 0 (*d*) $2f(x)$

(*iii*) If the function $f(x)$ is an odd function and $a = -b$, then the value of definite integral will be

(*a*) $2\int_0^b f(x)\,dx$ (*b*) 0 (*c*) $\int_0^b f(x)\,dx$ (*d*) $-\int_0^b f(x)\,dx$

(*iv*) If $f(x)$ is an even function and $a = -b$, then the value of definite integral will be

(*a*) $2\int_0^b f(x)\,dx$ (*b*) 0 (*c*) $\int_0^b f(x)\,dx$ (*d*) $-\int_0^b f(x)\,dx$

(*v*) If $a = 0$, then the definite integral will be

(*a*) $2\int_0^b f(x)\,dx$ (*b*) $\int_0^a f(b-x)\,dx$ (*c*) $\int_0^b f(b-x)\,dx$ (*d*) $-\int_0^b f(x)\,dx$

**Ans.** (*i*) (*b*), (*ii*) (*c*), (*iii*) (*b*), (*iv*) (*a*), (*v*) (*c*)

# 8 APPLICATION OF INTEGRALS

## Multiple Choice Questions

**1.** Area bounded by the curve $y = \sin x$ and the $x$-axis between $x = 0$ and $x = 2\pi$ is

(*a*) 2 sq units (*b*) 0 sq units (*c*) 3 sq units (*d*) 4 sq units

**Sol.** (*d*), as $\sin x$ is positive in 1st and 2nd quadrant and negative is 3rd and 4th quadrant.

$$\text{Area} = \int_0^{2\pi} |\sin x|\, dx = \int_0^{\pi} \sin x\, dx + \int_{\pi}^{2\pi} (-\sin x)\, dx = 4 \text{ sq units}$$

**2.** The area of the region bounded by the curve $y = \frac{1}{x}$, the $x$-axis and between $x = 1$ to $x = 6$ is

(*a*) $\frac{1}{36}$ sq units (*b*) $\frac{1}{6}$ sq units (*c*) $\log_e 6$ sq units (*d*) $-\log_e 6$ sq units

**Sol.** (*c*), as area $= \int_1^6 \frac{1}{x} dx = [\log|x|]_1^6 = \log_e 6$ sq units

**3.** The area of the region bounded by the curve $y = \frac{1}{x}$, $x$-axis and between $x = 1$, $x = 4$ is

(*a*) log 4 sq units (*b*) $\frac{1}{4}$ sq units (*c*) $\frac{1}{16}$ sq units (*d*) –log 4 sq units

**Sol.** (*a*), as curve is $y = \frac{1}{x}$, $x$-axis and between $x = 1$, $x = 4$

$$\text{Area} = \int_1^4 \frac{1}{x}\, dx = [\log|x|]_1^4 = \log 4 - \log 1 = \log 4 \text{ sq units.}$$

**4.** The area enclosed by the curve $y = x^2$ and $y = 8$ is

(*a*) $\frac{64\sqrt{2}}{3}$ sq units (*b*) $\frac{512}{3}$ sq units (*c*) $\frac{64}{3}$ sq units (*d*) $24\sqrt{2}$ sq units

**Sol.** (*a*), as area $= 2\int_0^8 \sqrt{y}\, dy = 2\cdot\left[\frac{2}{3}y^{3/2}\right]_0^8 = \frac{4}{3}(8)^{3/2} = \frac{4}{3}\cdot 8\cdot 2\sqrt{2}$ sq units $= \frac{64}{3}\sqrt{2}$ sq units

**5.** Area of the region bounded by the curve $y = \sqrt{49 - x^2}$ and the $x$-axis is

(*a*) $\frac{49}{2}\pi$ sq units (*b*) $98\pi$ sq units (*c*) $49\pi$ sq units (*d*) $240\pi$ sq units

**Sol.** (*a*), as area is above the $x$-axis

$$\therefore \quad \text{area} = 2\int_0^7 \sqrt{49 - x^2}\, dx = 2\left[\frac{x}{2}\sqrt{49 - x^2} + \frac{49}{2}\sin^{-1}\frac{x}{7}\right]_0^7$$

$$= 2\left[\left(\frac{7}{2}\times 0 + \frac{49}{2}\sin^{-1}1\right) - (0)\right] = \frac{49}{2}\pi \text{ sq units}$$

**6.** Area of the region bounded by the curve $x = 2y + 3$, the $y$-axis and between $y = -1$ and $y = 1$ is **[KVS]**

(*a*) 4 sq units (*b*) $\frac{3}{2}$ sq units (*c*) 6 sq units (*d*) 8 sq units

**Sol.** (*c*), as area = $\int_{-1}^{1}(2y+3)\,dy$ = 6 sq units

**7.** The area enclosed by the circle $x^2 + y^2 = 8$ is

(*a*) $16\pi$ sq units (*b*) $2\sqrt{2}\,\pi$ sq units (*c*) $8\pi^2$ sq units (*d*) $8\pi$ sq units

**Sol.** (*d*), as for circle $x^2 + y^2 = 8$, centre is (0, 0), radius = $\sqrt{8}$.

$\therefore$ Area = $4\int_{0}^{2\sqrt{2}}\sqrt{8-x^2}\,dx$ = $8\pi$ sq units

**8.** The area bounded between the curves $y^2 = 6x$ and $x^2 = 6y$ is

(*a*) 6 sq units (*b*) 12 sq units (*c*) 36 sq units (*d*) 24 sq units

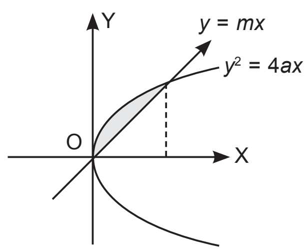
**Sol.** (*b*), as both the curves cut at $x = 0$ and $x = 6$

$\therefore$ Area = $\int_{0}^{6}\left(\sqrt{6}\,x - \frac{x^2}{6}\right)dx$ = 12 sq units

**9.** If the area bounded by the curves $y^2 = 4ax$ and $y = mx$ is $\frac{a^2}{3}$, then the value of $m$ is

(*a*) 2 (*b*) –2 (*c*) $\frac{1}{2}$ (*d*) none of these

**Sol.** (*a*),




$(mx)^2 = 4ax \Rightarrow m^2x^2 = 4ax \Rightarrow x = 0,\ x = \frac{4a}{m^2}$

as the two curves intersect at 0, $\frac{4a}{m^2}$

$$\therefore \quad \text{Area} = \int_{0}^{\frac{4a}{m^2}}\left(\sqrt{4ax} - mx\right)dx = \left[2\sqrt{a}\cdot\frac{2}{3}x^{3/2} - \frac{mx^2}{2}\right]_0^{\frac{4a}{m^2}}$$

$$= \frac{4\sqrt{a}}{3}\cdot\frac{4a}{m^2}\cdot\frac{2\sqrt{a}}{m} - \frac{m}{2}\cdot\frac{16a^2}{m^4} = \frac{32a^2}{3m^3} - \frac{8a^2}{m^3} = \frac{8a^2}{3m^3}$$

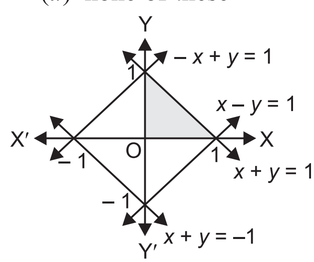
Given $\frac{8a^2}{3m^3} = \frac{a^2}{3} \Rightarrow m^3 = 8 \Rightarrow m = 2.$

**10.** The area enclosed within the curve $|\,x\,| + |\,y\,| = 1$ is

(*a*) 21 (*b*) 1.5 (*c*) 2 (*d*) none of these

**Sol.** (*c*), curves are $x + y = 1$, $-x - y = 1$,

$-x + y = 1$, $x - y = 1$




Area = 4 × area in 1st quadrant

$$= 4\int_{0}^{1}(1-x)dx = 4\left[x - \frac{x^2}{2}\right]_0^1$$

$$= 4\left(1 - \frac{1}{2}\right) - 0 = 2 \text{ sq units}$$

**11.** The area bounded by the curve $x^2 = 4y$ and the straight line $x = 4y - 2$ is

(*a*) $\frac{3}{8}$ (*b*) $\frac{5}{8}$ (*c*) $\frac{7}{8}$ (*d*) $\frac{9}{8}$

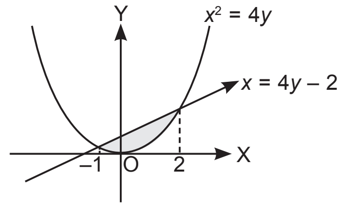
**Sol.** (*d*),

$\Rightarrow \quad x = x^2 - 2 \Rightarrow x^2 - x - 2 = 0$

$\Rightarrow \quad (x - 2)(x + 1) = 0 \Rightarrow x = -1, 2$

as the two curves intersect at –1, 2

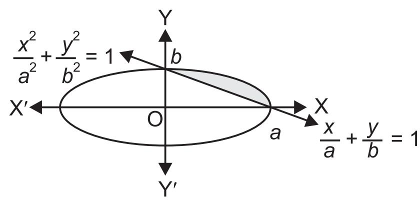
$\therefore$ Area $= \int_{-1}^{2}\left(\frac{x+2}{4} - \frac{x^2}{4}\right)dx = \frac{9}{8}$ sq units




**12.** The area of the smaller region between the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$ in first quadrant is

(*a*) $\frac{1}{2}ab$ sq units (*b*) $\frac{1}{2}\pi ab$ sq units (*c*) $\pi ab$ sq units (*d*) $\frac{ab}{4}(\pi - 2)$ sq units

**Sol.** (*d*), $y_1: \frac{x^2}{a^2} + \frac{y^2}{b^2} = 1;\ y_2 = \frac{x}{a} + \frac{y}{b} = 1$




$$\text{Area} = \int_0^a (y_1 - y_2)\,dx$$

$$= \int_0^a \left\{\frac{b}{a}\sqrt{a^2 - x^2} - \frac{b}{a}(a - x)\right\}dx$$

$$= \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} - ax + \frac{x^2}{2}\right]_0^a$$

$$= \frac{b}{a}\left[\left(0 + \frac{a^2}{2}\cdot\sin^{-1}1 - a^2 + \frac{a^2}{2}\right) - 0\right]$$

$$= \frac{b}{a}\left[\frac{a^2}{2}\cdot\frac{\pi}{2} - \frac{a^2}{2}\right] = \frac{b}{a}\cdot\frac{a^2}{2}\left(\frac{\pi}{2} - 1\right)$$

$$= \frac{ab}{4}(\pi - 2) \text{ sq units}$$

**13.** The area enclosed by the circle $x^2 + y^2 = 16$ is

(*a*) 20 sq units (*b*) $20\pi$ sq units (*c*) $16\pi$ sq units (*d*) $256\pi$ sq units

**Ans.** (*c*)

**14.** Area of the region in the first quadrant enclosed by the *x*-axis, the line $y = x$ and the circle $x^2 + y^2 = 32$ is

(*a*) $16\pi$ sq units (*b*) $4\pi$ sq units (*c*) $32\pi$ sq units (*d*) none of these

**Ans.** (*b*)

**15.** Area of the region bounded by the curve $y^2 = 4x$, *y*-axis and the line $y = 3$ is

(*a*) 2 sq units (*b*) $\frac{9}{4}$ sq units (*c*) $\frac{9}{3}$ sq units (*d*) $\frac{9}{2}$ sq units

**Ans.** (*b*)

**16.** Area of the region bounded by the curve $y = \cos x$ between $x = 0$ and $x = 2\pi$ is

(*a*) 4 sq units (*b*) 3 sq units (*c*) 2 sq units (*d*) 1 sq unit

**Ans.** (*a*)

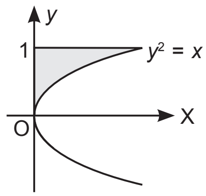
**17.** The area bounded by the curve $y^2 = x$, the $y$-axis and the line $y = 1$ is

(*a*) $\frac{1}{3}$ sq units (*b*) $\frac{1}{6}$ sq units (*c*) $\frac{2}{3}$ sq units (*d*) $\frac{1}{2}$ sq units

**Sol.** (*a*), area bounded by the curve $y^2 = x$, the $y$-axis and $y = 1$ is

$$A = \int_0^1 x\,dy = \int_0^1 y^2 dy$$

$$= \left[\frac{y^3}{3}\right]_0^1 = \frac{1}{3} \text{ sq units.}$$




**18.** The area bounded by the curve $y = \frac{2}{x}$, the $x$-axis and between $x = 2$ to $x = 4$ is

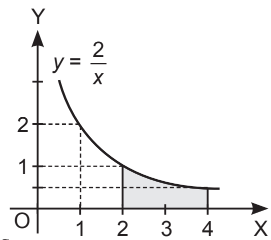
(*a*) 2 sq units (*b*) –2 sq units (*c*) log 2 sq units (*d*) 2 log 2 sq units

**Sol.** (*d*), as area bounded by the curve $y = \frac{2}{x}$, the $x$-axis and between $x = 2$ to 4 is

$$A = \int_2^4 y\,dx = \int_2^4 \frac{2}{x} dx$$

$$= 2\,[\log x]_2^4$$

$$= 2[\log 4 - \log 2] = 2 \log 2 \text{ sq units.}$$



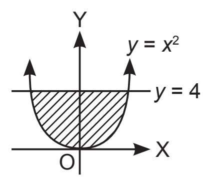

**19.** The area of the region bounded by the curve $y = x^2$ and the line $y = 4$ is

(*a*) $\frac{16}{3}$ sq units (*b*) $\frac{8}{3}$ sq units (*c*) $\frac{32}{3}$ sq units (*d*) 8 sq units

**Sol.** (*c*), as curve is symmetrical about the $y$-axis.

$$\therefore \quad \text{Area} = 2\int_0^4 x\,dy = 2\int_0^4 \sqrt{y}\ dy$$

$$= \frac{4}{3}[y^{3/2}]_0^4 = \frac{4}{3} \times (8 - 0) = \frac{32}{3} \text{ sq units}$$

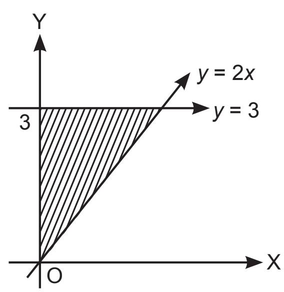



**20.** The area bounded by the line $y = 2x$, the $y$-axis and the line $y = 3$ is

(*a*) $\frac{9}{4}$ sq units (*b*) $\frac{9}{2}$ sq units (*c*) $\frac{9}{8}$ sq units (*d*) $\frac{9}{16}$ sq units

**Sol.** (*a*), as shaded area $= \int_0^3 x\,dy$

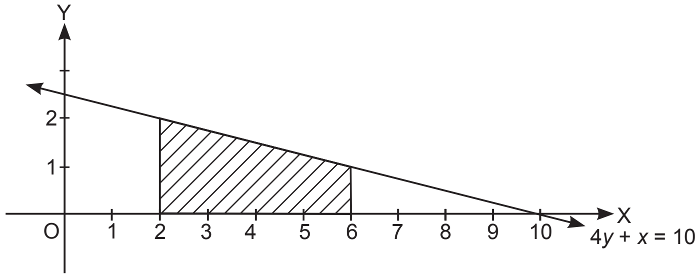
$$= \int_0^3 \frac{y}{2} dy = \left[\frac{1}{4}y^2\right]_0^3 = \frac{9}{4} \text{ sq units.}$$



**21.** The area of the region bounded by the line $4y + x = 10$, the $x$-axis and the lines $x = 2$ and $x = 6$ is

($a$) 6 sq units ($b$) 3 sq units ($c$) 12 sq units ($d$) 4.5 sq units

**Sol.** ($a$), as given line is $4y + x = 10 \Rightarrow x = 10 - 4y$




We have to find shaded area

$$A = \int_2^6 y\,dx = \int_2^6 \frac{10-x}{4}dx = \frac{1}{4}\left[10x - \frac{x^2}{2}\right]_2^6$$

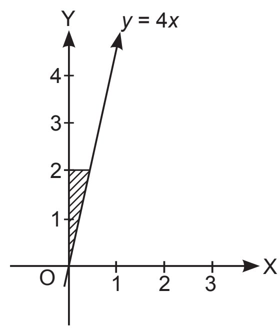
$$= \frac{1}{4}[(60-18)-(20-2)] = \frac{1}{4}(42-18) = \frac{1}{4}\times 24 = 6 \text{ sq units}$$

**22.** The area bounded by the line $y = 4x$, the $y$-axis and the line $y = 2$ is

($a$) 2 sq units ($b$) 4 sq units ($c$) $\frac{1}{4}$ sq units ($d$) $\frac{1}{2}$ sq units

**Sol.** ($d$), as

$$\text{area} = \int_0^2 x\,dy = \int_0^2 \frac{y}{4}dy = \left[\frac{1}{8}y^2\right]_0^2 = \frac{1}{8}\times(4-0) = \frac{1}{2} \text{ sq units}$$



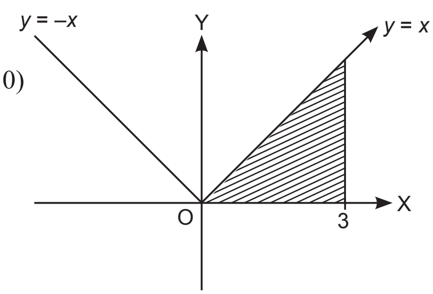

**23.** The area bounded by the curve $y = |x|$, $y = 0$ and $x = 3$ in first quadrant is

($a$) $\frac{9}{2}$ sq units ($b$) 9 sq units ($c$) $\frac{25}{2}$ sq units ($d$) 25 sq units

**Sol.** ($a$), as we have to find shaded area

$$\text{Area} = \int_0^3 |x|dx = \int_0^3 x\,dx \quad (\because |x| = x, x \geq 0)$$

$$= \left[\frac{x^2}{2}\right]_0^3 = \frac{9}{2} \text{ sq units}$$



**24.** The area bounded by $y = x^3$, the $x$-axis and $x = -1$ to $x = 1$ is

(*a*) $\frac{1}{4}$ sq units (*b*) $\frac{1}{2}$ sq units (*c*) $\frac{1}{3}$ sq units (*d*) $\frac{1}{5}$ sq units

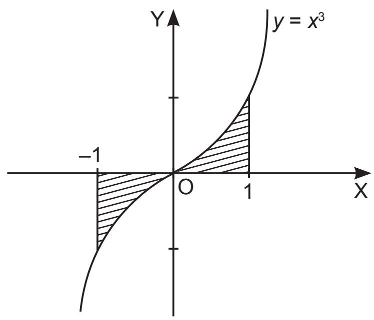
**Sol.** (*b*) as,

$$\text{area} = \left|\int_{-1}^{0} x^3 dx\right| + \int_{0}^{1} x^3 dx$$

$$= \left|\left[\frac{x^4}{4}\right]_{-1}^{0}\right| + \left[\frac{x^4}{4}\right]_{0}^{1}$$

$$= \left|0 - \frac{1}{4}\right| + \left(\frac{1}{4} - 0\right)$$

$$= \frac{1}{4} + \frac{1}{4} = \frac{1}{2} \text{ sq units}$$




**25.** The area of the region bounded by the parabola $y^2 = 16x$ and its latus rectum is

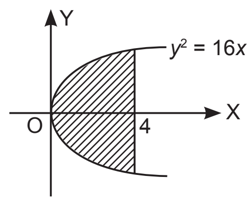
(*a*) $\frac{64}{3}$ sq units (*b*) $\frac{32}{3}$ sq units (*c*) 16 sq units (*d*) $\frac{128}{3}$ sq units

**Sol.** (*d*) Given parabola is $y^2 = 16x \Rightarrow 4a = 16 \Rightarrow a = 4$

$\therefore$ latus rectum is represented by $x = 4$

We have to find shaded area.

Curve is symmetrical to the $x$-axis.




$$\therefore \quad \text{Area} = 2\int_{0}^{4} y\, dx = 2\int_{0}^{4} 4\sqrt{x}\, dx$$

$$= \frac{8 \times 2}{3}[x^{3/2}]_{0}^{4}$$

$$= \frac{16}{3}[8 - 0] = \frac{128}{3} \text{ sq units}$$

**26.** The area bounded by the curve $y = |x|$, the $x$-axis and between $x = -2$ to $x = 0$ is

(*a*) 4 sq units (*b*) $\frac{3}{2}$ sq units (*c*) 1 sq unit (*d*) 2 sq units

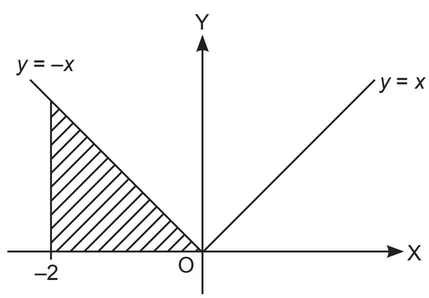
**Sol.** (*d*), as

$$\text{area} = \int_{-2}^{0} y\, dx = \int_{-2}^{0} |x| dx$$

$$= -\int_{-2}^{0} x\, dx$$

$$= -\left[\frac{x^2}{2}\right]_{-2}^{0} = -0 + \frac{4}{2}$$

$$= 2 \text{ sq units}$$



## Assertion-Reason Questions

**Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:**

(*a*) If both A and R are true and R is the correct explanation of A.

(*b*) If both A and R are true but R is not the correct explanation of A.

(*c*) A is true, R is false.

(*d*) A is false, R is true.

**27. Assertion (A):** Area enclosed by the ellipse $9x^2 + y^2 = 36$ and the line $3x + y = 6$ in first quadrant is $(3\pi - 6)$ square units.

**Reason (R):** area $= \int_0^2 \left(\sqrt{36 - 9x^2}\right) dx - \int_0^2 (6 - 3x)\,dx$

**Sol.** (*a*),

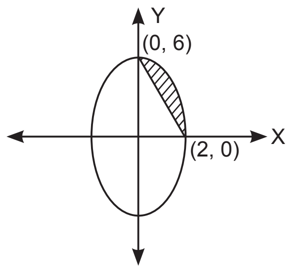
equation of ellipse is $9x^2 + y^2 = 36$ ...(*i*)

$$\Rightarrow \quad \frac{x^2}{4} + \frac{y^2}{36} = 1$$

equation of line is $3x + y = 6$ ...(*ii*)

$$\Rightarrow \quad \frac{x}{2} + \frac{y}{6} = 1$$

So area is represented by the shaded region




$$\begin{aligned}
\text{area} &= \int_0^2 y\,dx - \int_0^2 y\,dx \\
&= \int_0^2 \sqrt{36 - 9x^2}\,dx - \int_0^2 (6 - 3x)\,dx \\
&= 3\int_0^2 \sqrt{(4 - x^2)}\,dx - \int_0^2 (6 - 3x)\,dx \\
&= 3\left[\frac{x}{2}\sqrt{4 - x^2} + \frac{4}{2}\sin^{-1}\frac{x}{2}\right]_0^2 - \left[6x - \frac{3x^2}{2}\right]_0^2 \\
&= 3[0 + 2\sin^{-1} 1 - 0 - 0] - [(12 - 6) - 0] \\
&= 3 \times 2 \times \frac{\pi}{2} - 6 = (3\pi - 6) \text{ sq units}
\end{aligned}$$

A is true, R is also, true and the correct explanation of A

**28. Assertion (A):** The area bounded by the curve $|x| + |y| = 1$ is 2 sq units.

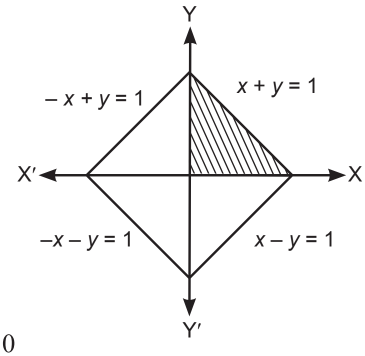
**Reason (R):** $\frac{d}{dx}|x| = \frac{|x|}{+x}$ where $x \neq 0$

**Sol.** (*b*)

curve is $|x| + |y| = 1$

$\Rightarrow \quad x + y = 1, \quad$ if $\quad x > 0, y > 0$

$\Rightarrow \quad x - y = 1, \quad$ if $\quad x > 0, y < 0$




$$\Rightarrow \quad -x + y = 1, \quad \text{if } x < 0, y > 0$$
$$-x - y = 1, \quad \text{if } x < 0, y < 0$$
$$\text{Area} = 4\int_0^1 y\,dx = 4\int_0^1 (1-x)\,dx$$
$$= 4\left[x - \frac{x^2}{2}\right]_0^1 = 2 \text{ sq units}$$

A is true.

$$\frac{d}{dx}|x|, \text{ if } x < 0, |x| = -x$$
$$\frac{d}{dx}(-x) = -1 \quad \text{for all } x < 0$$
$$\frac{|x|}{+x} = \frac{|x|}{x} = \frac{-x}{x} = -1$$
$$\frac{d|x|}{dx} = \frac{d}{dx}(x) = 1 \quad \text{for all } \frac{|x|}{x} = \frac{x}{x} = 1$$

So R is also true but not the correct explanation of A.

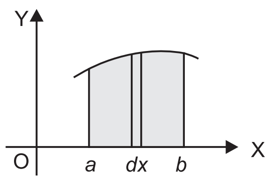
**29.** **Assertion (A):** If $f(x)$ is continuous and non negative in $[a, b]$, then the area of region bounded by the curve $y = f(x)$, $x$-axis and the ordinates $x = a$ and $x = b(b > a)$ is given by $\int_a^b f(x)\,dx$

**Reason (R):** If the function is differentiable in $(a, b)$ then it is continuous in $[a, b]$

**Ans.** (*b*), A and R both are true but R is not the correct explanation of A.

## Case-based Questions

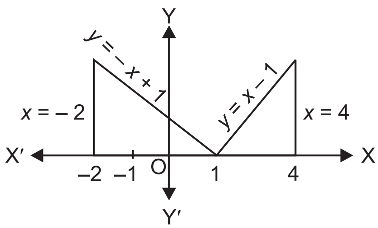
**30.** Area bounded by the curve $y = f(x)$, the $x$-axis and between the ordinates at $x = a$ and $x = b$ is given by area $= \int_a^b y\,dx = \int_a^b f(x)\,dx$



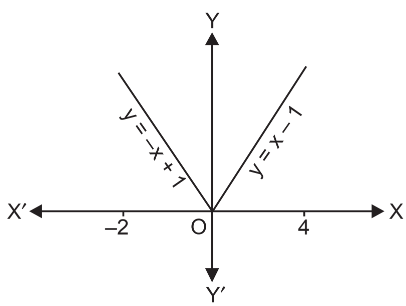

Based on the above information, answer the following:

(*i*) The graph of $y = |x - 1|$ is

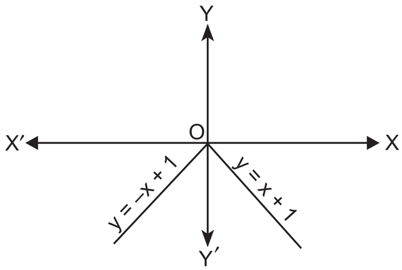
(*a*)



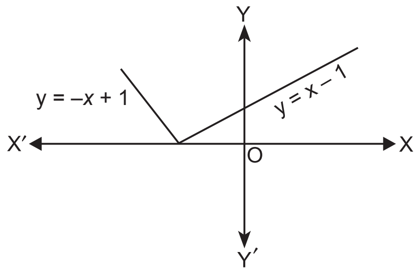

(*b*)



(*c*)




(*d*)




(*ii*) The value of $\int |x-1|\, dx$ is

(*a*) $\frac{|x+1|(x-1)}{3}+C$ (*b*) $\frac{|x-1|(x-1)}{2}+C$

(*c*) $\frac{(x-1)^2}{2}+C$ (*d*) $|x-1|+C$

(*iii*) The value of $\int_{-2}^{4} |x-1|\, dx$ is

(*a*) 4 sq units (*b*) 3 sq units (*c*) 9 sq units (*d*) 10 sq units

(*iv*) The coordinates of point at which the graph of $y = |x - 1|$ crosses $y$-axis is

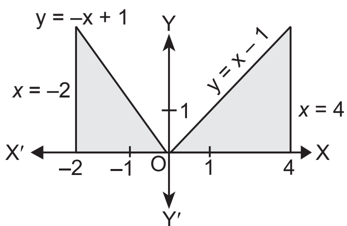
(*a*) (0, 2) (*b*) (0, 1) (*c*) (0, –1) (*d*) (0, 3)

(*v*) The value of the definite integral on the graph represents

(*a*) volume bounded by the curve (*b*) density bounded by the curve

(*c*) area bounded by the curve (*d*) length of the curve

**Sol.** (*i*) (*a*),




$y = |x - 1|$

$y = -x + 1$ or $y = x - 1$

(*ii*) (*b*), Let $|x-1| = \begin{cases} (x-1) & x \geq 1 \\ -(x-1) & x < 1 \end{cases}$

$$\int |x-1|\, dx = \int (x-1)\, dx \text{ or } -\int (x-1)\, dx = \frac{(x-1)^2}{2}+C \text{ or } \frac{-(x-1)^2}{2}+C$$

$$= \frac{(x+1)(x-1)}{2} \text{ or } \frac{-(x-1)(x-1)}{2} = \frac{|x-1|(x-1)}{2}.$$

(*iii*) (*c*), $\int_{-2}^{4} |x-1|\, dx = \int_{-2}^{1} -(x-1)\, dx + \int_{1}^{4} (x-1)\, dx$

$$= \left[\frac{-x^2}{2}+x\right]_{-2}^{1} + \left[\frac{x^2}{2}-x\right]_{1}^{4}$$

$$= \left(-\frac{1}{2}+1\right) - \left(-\frac{4}{2}-2\right) + \left(\frac{16}{2}-4\right) - \left(\frac{1}{2}-1\right) = 9 \text{ sq units.}$$

(*iv*) (*b*), We have $y = \begin{cases} x-1, & x \geq 1 \\ -x+1, & x < 1 \end{cases}$

When $x = 0 \Rightarrow y = -0 + 1 = 1$, point is (0, 1)

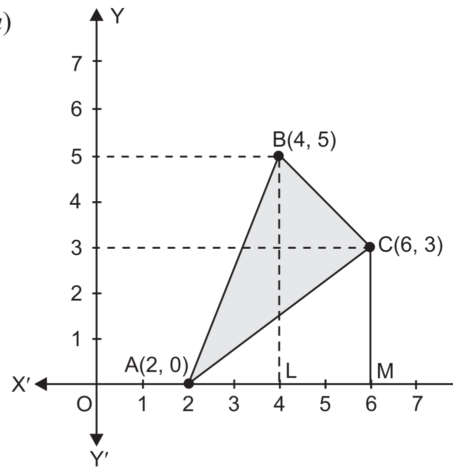
(*v*) (*c*), The value of the integral represents the area bounded by the curve, the *x*-axis and between ordinates at $x = a$, $x = b$.

**31.** Due to COVID-19, three students A, B and C are sitting in the class at (2, 0), (4, 5) and (6, 3) respectively while teacher is at (0, 0).

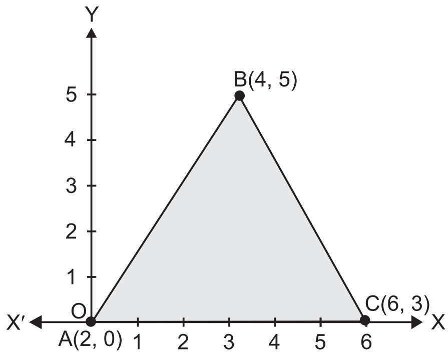
Based on the above information, answer the following

(*i*) The graph of sitting arrangement of the students is

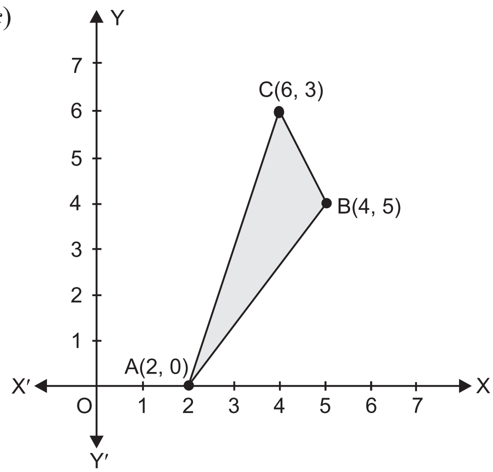
(*a*)



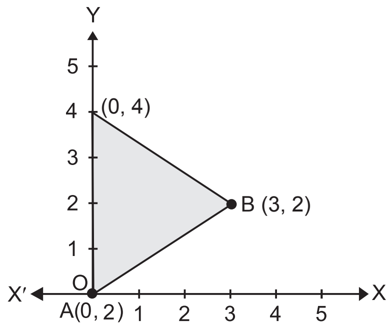

(*b*)




(*c*)




(*d*)



(*ii*) The equation of AB of the graph is

(*a*) $y = \frac{5}{2}(x + 2)$ (*b*) $x = \frac{5}{2}(y + 3)$

(*c*) $y = \frac{5}{2}(x - 2)$ (*d*) $x = \frac{5}{2}(y - 3)$

(*iii*) The equation of BC of the graph is

(*a*) $y = -x + 9$ (*b*) $y = x + 9$

(*c*) $y = x - 9$ (*d*) $y = -x - 9$

(*iv*) The equation of CA of the graph is

(*a*) $y = \frac{3}{2}(x - 2)$ (*b*) $y = \frac{3}{2}(x + 2)$

(*c*) $y = \frac{3}{4}(x - 2)$ (*d*) $y = \frac{3}{4}(x + 2)$

(*v*) The area of the graph by using integration method is

(*a*) 5 sq units (*b*) 6 sq units

(*c*) 7 sq units (*d*) 9 sq units

**Sol.** (*i*) (*a*), (*ii*) (*c*), (*iii*) (*c*), (*iv*) (*c*) , (*v*) (*c*)

# 9 DIFFERENTIAL EQUATIONS

## Multiple Choice Questions

**1.** Order of differential equation corresponding to family of curves $y = Ae^{2x} + Be^{-2x}$ is

(*a*) 2 (*b*) 1 (*c*) 3 (*d*) 4

**Sol.** (*a*), as there are two arbitrary constants and we have to differentiate twice.

**2.** The order of the differential equation corresponding to the family of curves $y = c(x - c)^2$, $c$ is constant is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) does not exist

**Sol.** (*a*), as there is one arbitrary constant.

**3.** If $p$ and $q$ are the degree and order of the differential equation $\left(\frac{d^2y}{dx^2}\right)^2 + 3\frac{dy}{dx} + \frac{d^3y}{dx^3} = 4$, then the value of $2p - 3q$ is

(*a*) 7 (*b*) –7 (*c*) 3 (*d*) –3

**Sol.** (*b*), as degree $p = 1$ and order $q = 3$

$$\therefore\ 2p - 3q = 2 - 9 = -7$$

**4.** The degree of the differential equation $\left(1+\frac{dy}{dx}\right)^3 = \left(\frac{dy}{dx}\right)^2$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

**Sol.** (*c*), as differential equation is

$$1 + 3\frac{dy}{dx} + 3\left(\frac{dy}{dx}\right)^2 + \left(\frac{dy}{dx}\right)^3 = \left(\frac{dy}{dx}\right)^2.$$ Exponent of the highest order derivative is 3.

**5.** The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) not defined

**Sol.** (*d*), as equation cannot be represented as a polynomial of derivatives.

**6.** General solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 2x + y$ is

(*a*) $e^{-y} = \frac{1}{2}e^{2x} + C$ (*b*) $\frac{1}{e^y} + \frac{1}{2}e^{2x} + C$ (*c*) $-e^{-y} = \frac{1}{2}e^{2x} + C$ (*d*) $e^{y} = \frac{1}{2}e^{2x} + C$

**Sol.** (*c*), as $\frac{dy}{dx} = e^{2x+y} = e^{2x} \cdot e^{y}$

$\Rightarrow \int e^{-y} dy = \int e^{2x} dx \Rightarrow - e^{-y} = \frac{1}{2} e^{2x} + C$

**7.** The particular solution of the differential equation $\frac{dy}{dx} = y \tan x$, given that $y = 1$ when $x = 0$ is

(*a*) $y = \cos x$ (*b*) $y = \sec x$ (*c*) $y = \tan x$ (*d*) $y = \sec x \tan x$

**Sol.** (*b*), as $\int \frac{dy}{y} = \int \tan x \, dx \Rightarrow \log |y| = \log|\sec x| + \log C$

$\Rightarrow \quad y = C \sec x$ ...(*i*)

Given $y = 1, x = 0 \Rightarrow 1 = C \sec 0 \Rightarrow C = 1$

$\therefore$ solution is $y = \sec x$ [from (*i*)]

**8.** Integrating factor for the solution of differential equation $\frac{dy}{dx} + 2y \tan x = \sin x$ is

(*a*) $\sec^2 x$ (*b*) $\sec x$ (*c*) $\log |\sec x|$ (*d*) $\tan x$

**Sol.** (*a*), Equation is $\frac{dy}{dx} + (2 \tan x) \cdot y = \sin x$

Integrating factor $= e^{\int 2 \tan x \, dx} = e^{2\int \tan x \, dx} = e^{2 \log |\sec x|} = \sec^2 x$

**9.** $F(x, y) = \frac{\sqrt{x^2 + y^2} + y}{x}$ is a homogeneous function of degree

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

**Sol.** (*a*), as if we find $F(\lambda x, \lambda y)$, we get $\lambda^{\circ} F(x, y)$

**10.** Differential equation representing the family of curves $(x + a)^2 + 2y^2 = a^2$ is of order

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) none of these

**Sol.** (*a*), as there is one arbitrary constant, and we have to differentiate once to eliminate *a*.

**11.** $y = e^{-x} + ax + b$ is a solution of differential equation

(*a*) $e^{-x} y'' = 1$ (*b*) $e^{x} y'' = 1$ (*c*) $e^{x} (y')^2 = 1$ (*d*) $e^{-x} (y')^2 = 1$

**Sol.** (*b*), as $y' = -e^{-x} + a$ and $y'' = e^{-x} \Rightarrow e^{x} y'' = 1$

**12.** $y = e^{m \cos^{-1} x}$ is a solution of differential equation

(*a*) $\sqrt{1 - x^2}\, y' = my$ (*b*) $(1 - x^2) y'' + xy' - m^2 y = 0$

(*c*) $(1 - x^2) y'' - xy' - m^2 y = 0$ (*d*) $(1 - x^2) y'' - xy' + m^2 y = 0$

**Sol.** (*c*), as if we find $y'$ and $y''$ and substitute in (*c*) it satisfies.

**13.** Degree of differential equation $t^2 \frac{d^2 s}{dt^2} - st \left(\frac{ds}{dt}\right)^2 = 5$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) none of these

**Sol.** (*a*), as exponent of the highest order derivative is 1.

**14.** For the solution of differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$, the integrating factor is

(*a*) $\frac{y}{x}$ (*b*) $x$ (*c*) $\frac{1}{x}$ (*d*) $-x$

**Sol.** (*b*)

**15.** Degree of differential equation $\left(\frac{d^3y}{dx^3}\right)^{\frac{2}{3}} = x$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) does not exist

**Sol.** (*b*)

**16.** Differential equation $e^x\frac{dy}{dx} = 3y^3$ can be solved using the method of

(*a*) separating the variables (*b*) homogeneous equations

(*c*) linear differential equation of first order (*d*) none of these

**Sol.** (*a*)

**17.** General solution of differential equation $\frac{dy}{dx} = x^5 + x^3 - \frac{2}{x}$ is

(*a*) $y = \frac{x^6}{6} + \frac{x^4}{4} - 2 \log |x|$ (*b*) $y = \frac{x^6}{6} + \frac{x^4}{4} - 2 \log |x| + 1$

(*c*) $y = 5x^4 + 3x^2 + \frac{2}{x^2} + C$ (*d*) $y = \frac{x^6}{6} + \frac{x^4}{4} - 2 \log |x| + C$

**Sol.** (*d*)

**18.** Differential equation $x\frac{dy}{dx} = y (\log y - \log x + 1)$ can be solved using the method of

(*a*) separating the variables (*b*) homogeneous equations

(*c*) linear differential equation of first order (*d*) none of these

**Sol.** (*b*)

**19.** The order of differential equation $y = \frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^3}$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) none of these

**Sol.** (*a*)

**20.** Degree of the differential equation $\frac{d^2y}{dx^2} + \sin\left(\frac{dy}{dx}\right) = 0$ is

(*a*) 2 (*b*) 1 (*c*) 0 (*d*) not defined

**Sol.** (*d*), as equation cannot be written as polynomial of differentials.

**21.** Integrating factor of the differential equation $\frac{dy}{dx} = x + y$ is

(*a*) –1 (*b*) 1 (*c*) $e^{-x}$ (*d*) $e^{-y}$

**Sol.** (*c*), as $\frac{dy}{dx} = x + y \Rightarrow \frac{dy}{dx} - y = x$

$$\text{I.F.} = e^{\int(-1)dx} = e^{-x}$$

**22.** The integrating factor for the differential equation $\frac{dy}{dx} + y \tan x - \sec x = 0$ is

(*a*) $\tan x$ (*b*) $\sec^2 x$ (*c*) $\sec x$ (*d*) $\frac{\tan^2 x}{2}$

**Sol.** (*c*), as integrating factor $= e^{\int \tan x\, dx} = e^{\log|\sec x|} = \sec x$

**23.** The sum of order and degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^3 = e^x$ is

(*a*) 2 (*b*) 3 (*c*) 5 (*d*) 4

**Sol.** (*b*), as given equation is $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^3 = e^x$

Order = 2, degree = 1

$\therefore$ sum = 2 + 1 = 3

**24.** If $p$ and $q$ are degree and order of a differential equation $\frac{dy}{dx} + \frac{1}{\frac{dy}{dx}} = 9$, then $2p + q$ is

(*a*) 5 (*b*) 4 (*c*) 3 (*d*) 7

**Sol.** (*a*), as given equation is $\frac{dy}{dx} + \frac{1}{\frac{dy}{dx}} = 9 \Rightarrow \left(\frac{dy}{dx}\right)^2 - 9\frac{dy}{dx} + 1 = 0$

order $q = 1$, degree $p = 2$

$\therefore 2p + q = 4 + 1 = 5$

## Assertion-Reason Questions

**Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:**

(*a*) If both A and R are true and R is the correct explanation of A.

(*b*) If both A and R are true but R is not the correct explanation of A.

(*c*) A is true, R is false.

(*d*) A is false, R is true.

**25. Assertion (A):** The order and degree of differential equation $y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$ is 1 and 2 respectively.

**Reason (R):** order and degree (if defined) of a differential equation are always positive integer.

**Sol.** (*a*)

$$y = x\frac{dy}{dx} + a\sqrt{1+\left(\frac{dy}{dx}\right)^2}$$

$$\Rightarrow \quad \left(y - x\frac{dy}{dx}\right)^2 = a^2\left[1+\left(\frac{dy}{dx}\right)^2\right]$$

$$\Rightarrow \quad y^2 + x^2\left(\frac{dy}{dx}\right)^2 - 2xy\frac{dy}{dx} = a^2 + a^2\left(\frac{dy}{dx}\right)^2$$

So order = 1, degree = 2

A is true, R is also true and the correct explanation of A.

**26. Assertion (A):** The order of differential equation of family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is 1.

**Reason (R):** The number of arbitrary constant in the general solution of a differential equation of order $n$ is always $n$.

**Sol.** (*d*), As we know that number of arbitrary constant in general solution = order of differential equation

So order = 2 (as $a$ and $b$ are two arbitrary constants, given)

So, A is false, but R is true.

**27. Assertion (A):** The degree of homogeneous function $F(x, y) = \frac{\sqrt{x^2+y^2}+y}{x}$ is 0.

**Reason (R):** If $F(\lambda x, \lambda y) = \lambda^n F(x, y)$ then $F(x, y)$ is called homogeneous function of degree $n$.

**Sol.** (*a*), as

$$F(x, y) = \frac{\sqrt{x^2+y^2}+y}{x}$$

$$F(\lambda x, \lambda y) = \frac{\sqrt{\lambda^2x^2+\lambda^2y^2}+\lambda y}{\lambda x}$$

$$= \frac{\lambda\left(\sqrt{x^2+y^2}+y\right)}{\lambda x} = \lambda^{\circ}F(x, y)$$

So, $F(x, y)$ is homogeneous function of degree 0.

A is true and R is also true and the correct explanation of A.

**28. Assertion (A):** The general solution of the differential equation $\frac{y}{x}\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is $(1 + y^2) = A(1 + x^2)$

**Reason (R):** $\int \frac{x}{1+x^2}dx = \frac{1}{2}\log|1+x^2| + C$

**Sol.** (*a*), as

$$\frac{y}{x}\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$$

Integrating both sides, we get

$$\int \frac{y}{1+y^2}dy = \int \frac{x}{1+x^2}dx$$

or

$$\frac{1}{2}\int \frac{2y}{1+y^2}dy = \frac{1}{2}\int \frac{2x}{1+x^2}dx$$

$$\Rightarrow \quad \frac{1}{2}\log|1+y^2| = \frac{1}{2}\log|1+x^2| + \log c$$

$$\Rightarrow \quad \frac{1}{2}\left[\log|1+y^2| - \log|1+x^2|\right] = \log C$$

$$\Rightarrow \quad \log\left|\frac{1+y^2}{1+x^2}\right| = 2\log C$$

$$\Rightarrow \quad \frac{1+y^2}{1+x^2} = C^2$$

let

$$C^2 = A \text{ (constant)}$$

$$(1 + y^2) = A(1 + x^2)$$

A is true and R is also true and the correct explanation of A.

**29. Assertion (A):** Solution of the differential equation

$$\frac{dy}{dx} = e^{3x-2y} + x^2 e^{-2y} \text{ is } \frac{e^{2y}}{3} = \frac{e^{3x}}{3} + \frac{x^2}{2} + C$$

**Reason (R):** $\int e^{kx}dx = \frac{e^{kx}}{k} + C$

**Sol.** (*d*), as

$$\frac{dy}{dx} = \frac{e^{3x}}{e^{2y}} + \frac{x^2}{e^{2y}}$$

Integrating both sides, we get

$$\int e^{2y}dy = \int (e^{3x} + x^2)dx$$

$$\Rightarrow \quad \frac{e^{2y}}{2} = \frac{e^{3x}}{3} + \frac{x^3}{3} + C$$

So A is false, but R is true.

## Case-based Questions

**30.** Polio drops are delivered to 50K children in a district. The rate at which polio drops are given is directly proportional to the number of children who have not been administered the drops. By the end of 2nd week half the children have been given the polio drops. How many will have been given the drops by the end of 3rd week can be estimated using the solution to the differential equation $\frac{dy}{dx} = k(50 - y)$ where $x$ denotes the number of weeks and $y$ the number of children who have been given the drops? **[CSBE Question Bank]**

(*i*) The order of the above given differential equation is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

(*ii*) Which method of solving a differential equation can be used to solve $\frac{dy}{dx} = k(50 - y)$?

(*a*) Variable separable method

(*b*) Solving homogeneous differential equation

(*c*) Solving linear differential equation

(*d*) all of the above

(*iii*) The solution of the differential equation $\frac{dy}{dx} = k(50 - y)$ is given by,

(*a*) $\log|50 - y| = kx + C$ (*b*) $-\log|50 - y| = kx + C$

(*c*) $\log|50 - y| = \log|kx| + C$ (*d*) $50 - y = kx + C$

(*iv*) The value of $c$ in the particular solution given that $y(0) = 0$ and $k = 0.049$ is

(*a*) $\log 50$ (*b*) $\log\frac{1}{50}$ (*c*) 50 (*d*) –50

(*v*) Which of the following solutions may be used to find the number of children who have been given the polio drops?

(*a*) $y = 50 - e^{kx}$ (*b*) $y = 50 - e^{-kx}$

(*c*) $y = 50(1 - e^{-kx})$ (*d*) $y = 50(e^{-kx} - 1)$

**Sol.** (*i*) (*a*) Order is 1

(*ii*) (*a*), Variable separable method

(*iii*) (*b*), $-\log|50 - y| = kx + C$

(*iv*) (*b*), $\log\frac{1}{50}$

(*v*) (*c*), $y = 50(1 - e^{-kx})$

**31.** An equation involving variables as well as derivative of the dependent variable with respect to only one independent variable is called an ordinary differential equation.

e.g. $$\frac{dy}{dx} + \frac{d^2y}{dx^2} - 2 = 0$$

From any given relation between the dependent and independent variables, a differential equation can be formed by differentiating it with respect to the independent variable and eliminating arbitrary constants involved.

(*i*) The degree of the differential equation $\left(\frac{dy}{dx}\right)^4 + 3y\frac{d^2y}{dx^2} = 0$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) 4

(*ii*) The order of differential equation $(y''')^2 + (y'')^3 + (y')^4 + y^5 = 0$ is

(*a*) 1 (*b*) 2 (*c*) 4 (*d*) 3

(*iii*) The number of arbitrary constants in the general solution of a differential equation of third order is

(*a*) 0 (*b*) 2 (*c*) 3 (*d*) 4

(*iv*) The degree of differential equation $x^3\left(\frac{d^2y}{dx^2}\right)^2 + x\left(\frac{dy}{dx}\right)^4 = 0$ is

(*a*) 1 (*b*) 0 (*c*) 4 (*d*) is not defined

**Sol.** (*i*) (*a*), The highest order derivative is $\frac{d^2y}{dx^2}$ whose degree is 1. So, degree of differential equation is 1.

(*ii*) (*d*), The highest order derivative of the differential equation is $y'''$.

Therefore, its order is 3.

(*iii*) (*c*), We know that the number of arbitrary constant in the general solution of a differential equation of order $n$ is equal to its order.

Therefore, the number of constants in the general equation of the third-order differential equation is three.

(*iv*) (*d*), as differential equation can't be written as polynomial of derivatives.

**32.** A function of the form $y = \phi(x) + C$, which satisfies given differential equation, is called the solution of the differential equation. The solution which contains as many arbitrary constants as the order of the differential equation, is called the general solution of the differential equation.

A solution obtained by giving particular values to arbitrary constants in the general solution of a differential equation, is called the particular solution.

(*i*) $y^2 = 4ax$ is solution of differential equation

(*a*) $yy' = xyy'' + xy'^2$ (*b*) $xy' = xyy' + xy''$

(*c*) $2xy' - y = 0$ (*d*) $xy'' + y' = xy$

(*ii*) The solution of the differential equation $\sec^2 x \cdot \tan y \, dx + \sec^2 y \tan x \, dy = 0$ is

(*a*) $\tan x = C$ (*b*) $\tan x \cdot \tan y = C$

(*c*) $\tan x + \tan y = C$ (*d*) $\frac{\tan x}{\tan y} = C$

(*iii*) The differential equation of the family of circles having centre on the $y$-axis and radiing 2 units is

(*a*) $x^2 + \frac{x^2}{y'} = 8$ (*b*) $x^2 + \frac{x^2}{y'} = 2$

(*c*) $x^2 + \frac{x^2}{y'} = 4$ (*d*) $x^2 + y' = 2$

(*iv*) The general solution of the differential equation $\frac{dy}{dx} = \frac{1+y^2}{1+x^2}$ is

(*a*) $\frac{\tan^{-1} y}{\tan^{-1} x} = C$ (*b*) $\tan^{-1} x \cdot \tan^{-1} y = C$

(*c*) $\frac{\tan^{-1} x}{\tan^{-1} y} = C$ (*d*) $\tan^{-1} y = \tan^{-1} x + C$

(*v*) The solution of differential equation $e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$ is

(*a*) $\tan x = C(1 - e^y)$ (*b*) $\tan y = C(1 - e^y)$

(*c*) $\tan y = C(1 - e^x)$ (*d*) $\tan y = C(1 - e^{-x})$

**Sol.** (*i*) (*c*), (*ii*) (*b*), (*iii*) (*c*), (*iv*) (*d*), (*v*) (*c*)

**33.** Two friends together are preparing for board examination and they were revising differential equations. They were asking each other one by one concepts related to differential equations, then one of the friends asked how to solve the differential equation, $(2x - 5y + 3)dx + (4x - 10y - 9)dy = 0$. During conversation they came across the following questions:

(*i*) Differential equations can be solved using

(*a*) seperating the variables

(*b*) method for solving homogeneous equations

(*c*) method for solving linear differential equations of first order

(*d*) using substitution method

(*ii*) We can start with

(*a*) separating variables (*b*) substituting $y = vx$

(*c*) finding integrating factor (*d*) substituting $2x - 5y = t$

(*iii*) $\int \frac{2x-9}{3x-1} dx$ is equal to

(*a*) $\frac{2}{3} - \frac{25}{9}\log|3x - 1| + C$ (*b*) $\frac{2}{3}x - \frac{25}{9}\log|3x - 1| + C$

(*c*) $\frac{2}{3} - \frac{25}{3}\log|3x - 1| + C$ (*d*) $\frac{2}{3}x - \frac{25}{3}\log|3x - 1| + C$

(*iv*) $\int 3 \cdot dx$ is equal to

(*a*) $3 + C$ (*b*) $\log 3 + C$

(*c*) $3x + C$ (*d*) $\frac{3^2}{2} + C$

(*v*) Solution of differential equation is

(*a*) $\frac{2}{9}(2x-5y)-\frac{25}{21}\log|18x-45y-3| = x + C$

(*b*) $(2x-5y)-\log|18x-45y-3| = x + C$

(*c*) $\frac{2}{9}(2x-5y)-\log|18x-45y-3| = x + C$

(*d*) $\frac{2}{9}(2x-5y)-\frac{25}{27}\log|18x-45y-3| = C$

**Sol.** (*i*) (*d*)

(*ii*) (*d*)

(*iii*) (*b*), as $\int \frac{2x-9}{3x-1}dx = \int \left\{\frac{2}{3}-\frac{\frac{25}{3}}{3x-1}\right\}dx$

$$= \frac{2}{3}x - \frac{25}{3}\cdot\frac{\log|3x-1|}{3} + C$$

$$= \frac{2}{3}x - \frac{25}{9}\log|3x-1| + C$$

$$3x-1\overline{\big)\,2x-9\,}\left(\frac{2}{3}\right.\qquad \begin{array}{r} 2x-\frac{2}{3} \\ -\quad + \\ \hline -\frac{25}{3} \end{array}$$

(*iv*) (*c*), $\int 3dx = 3\int 1\cdot dx = 3x + C$

(*v*) (*a*), Consider equation $(2x-5y+3)dx + (4x-10y-9)dy = 0$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{(2x-5y+3)}{(4x-10y-9)}$$

$$\Rightarrow \quad 5\frac{dy}{dx} = \frac{-5(2x-5y+3)}{4x-10y-9}$$

$$\Rightarrow \quad 2-\frac{dt}{dx} = \frac{-5(t+3)}{2t-9}$$

$$\Rightarrow \quad \frac{dt}{dx} = 2+\frac{5t+15}{2t-9} = \frac{4t-18+5t+15}{2t-9} = \frac{9t-3}{2t-9}$$

$$\Rightarrow \quad \frac{2t-9}{9t-3}dt = dx$$

Let $2x-5y = t$

$\Rightarrow 2-5\frac{dy}{dx} = \frac{dt}{dx}$

$\Rightarrow 2-\frac{dt}{dx} = 5\frac{dy}{dx}$

On integrating both sides, we get

$$\int\left(\frac{2}{9}-\frac{25/3}{9t-3}\right)dt = \int dx$$

$$\Rightarrow \quad \frac{2}{9}t - \frac{25}{3\times 9}\log|9t-3| = x + C$$

$$\Rightarrow \frac{2}{9}(2x-5y)-\frac{25}{27}\log|18x-45y-3| = x + C$$ is required solution.

$$9t-3\overline{\big)\,2t-9\,}\left(\frac{2}{9}\right.\qquad \begin{array}{r} 2t-\frac{2}{3} \\ -\quad + \\ \hline -\frac{25}{3} \end{array}$$

**34.** Students are preparing for their boards and are revising differential equations, one of the questions taken up is consider the differential equation $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$.

Based on the above answer the following:

(*i*) The order of the differential equation is

(*a*) 6 (*b*) 3 (*c*) 1 (*d*) 2

(*ii*) The given differential equation represents an equation

(*a*) in which variables can be seperated.

(*b*) which is homogeneous.

(*c*) which is linear differential equation of first order.

(*d*) which is linear differential equation of second order.

(*iii*) To show that the given equation is homogeneous we show that

(*a*) $\frac{dy}{dx} = f(x, y)$ (*b*) $\frac{dy}{dx} = f(x) \cdot f(y)$

(*c*) $\frac{dy}{dx} = f\left(\frac{y}{x}\right)$ (*d*) $\frac{dy}{dx} + P(x) \cdot y = Q(x)$

(*iv*) To solve the given differential equation, we substitutes

(*a*) $y = v^2x$ (*b*) $y = vx^2$

(*c*) $y^2 = vx$ (*d*) $y = vx$

(*v*) The solution of given differential equation is

(*a*) $(x^2 - y^2) = C(x^2 + y^2)^2$, $C$ is constant

(*b*) $(x^2 - y^2)^2 = C(x^2 + y^2)$, $C$ is a constant

(*c*) $(x^2 + 2y^2)^2 = C(2x^2 - y^2)$, $C$ is a constant

(*d*) $(x^2 - 2y^2)^2 = C(2x^2 + y^2)$, $C$ is a constant

**Sol.** (*i*) (*c*), Consider equation, $(x^3 - 3xy^2)dx = (y^3 - 3x^2y)dy$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{x^3 - 3xy^2}{y^3 - 3x^2y} \quad \text{...}(i)$$

order = 1

(*ii*) (*b*)

(*iii*) (*c*)

(*iv*) (*d*)

(*v*) (*a*) $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

From (*i*),

$$v + x\frac{dv}{dx} = \frac{x^3 - 3x^3v^2}{x^3v^3 - 3x^3v} = \frac{1 - 3v^2}{v^3 - 3v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1-3v^2}{v^3-3v} - v = \frac{1-3v^2-v^4+3v^2}{v^3-3v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{1-v^4}{v^3-3v} \Rightarrow \int \frac{v^3-3v}{1-v^4}dv = \int \frac{dx}{x}$$

$$\Rightarrow \quad \int \frac{v^3}{1-v^4}dv - 3\int \frac{v}{1-v^4}dv = \int \frac{dx}{x} \qquad ...(ii)$$

For $\int \frac{v^3}{1-v^4}dv = \frac{1}{4}\int \frac{1}{1-t}dt = -\frac{1}{4}\log|1-t|$ $\quad \Big|$ Let $v^4 = t \Rightarrow 4v^3dv = dt$

$$= -\frac{1}{4}\log|1-v^4|$$

For $\int \frac{v}{1-v^4}dv = \frac{1}{2}\int \frac{1}{1-t^2}dt$ $\quad \Big|$ Let $v^2 = t \Rightarrow 2vdv = dt$

$$= \frac{1}{2\times 2}\log\left|\frac{1+t}{1-t}\right| = \frac{1}{4}\log\left|\frac{1+v^2}{1-v^2}\right|$$

From (*ii*), we get

$$-\frac{1}{4}\log|1-v^4| - \frac{3}{4}\log\left|\frac{1+v^2}{1-v^2}\right| = \log|x| + \log|k|$$

$$\Rightarrow \quad -\left[\frac{1}{4}\log|1-v^4| + \frac{3}{4}\log\left|\frac{1+v^2}{1-v^2}\right|\right] = -\log\left|\frac{1}{xk}\right|$$

$$\Rightarrow \quad \frac{1}{4}\left[\log|1-v^4| + 3\log\left|\frac{1+v^2}{1-v^2}\right|\right] = \log\left|\frac{1}{xk}\right|$$

$$\Rightarrow \quad \log\left|\frac{(1-v^2)(1+v^2)\times(1+v^2)^3}{(1-v^2)^3}\right| = \log\left|\frac{1}{xk}\right|^4$$

$$\Rightarrow \quad \frac{(1+v^2)^4}{(1-v^2)^2} = \frac{1}{x^4k^4}$$

$$\Rightarrow \quad (1+v^2)^2 = \frac{(1-v^2)}{x^2k^2}$$

$$\Rightarrow \quad \left(1+\frac{y^2}{x^2}\right)^2 x^2k^2 = \left(1-\frac{y^2}{x^2}\right)$$

$$\Rightarrow \quad \frac{(x^2+y^2)^2}{x^4}\cdot x^2k^2 = \frac{(x^2-y^2)}{x^2}$$

$\Rightarrow \quad (x^2 + y^2)^2C = (x^2 - y^2)$ (where $k^2 = C$, constant)

**35.** As some topic need revision for examinations one such topic is integration and it's use in solving differential equations. Friends have one differential equation before them as

$x \cos y\, dy = (xe^x \log x + e^x)dx$ and they are trying to answer the following, help them

(*i*) The degree of the differential equation is

(*a*) not defined (*b*) 1 (*c*) 0 (*d*) 2

(*ii*) The above equation can be solved using

(*a*) method of separating the variables

(*b*) method of solving homogeneous equations

(*c*) method of solving linear differential equation of first order

(*d*) by substitution

(*iii*) $\int \cos y\, dy$ is

(*a*) $\sin y + C$ (*b*) $-\sin y + C$ (*c*) $\sec y + C$ (*d*) $-\sec y + C$

(*iv*) $\int \left(\frac{xe^x \log x + e^x}{x}\right) dx$ is

(*a*) $\log x + C$ (*b*) $e^x + C$ (*c*) $\frac{e^x}{\log x} + C$ (*d*) $e^x \log x + C$

(*v*) Solution of differential equation is

(*a*) $\sin y + e^x \log x = 0$ (*b*) $\sin x = e^x \log y + C$

(*c*) $\sin y = e^x \log x + C$ (*d*) $\sin y = e^x \log x + 1$

**Sol.** (*i*) (*b*), (*ii*) (*a*), (*iii*) (*a*), (*iv*) (*d*), (*v*) (*c*)

**36.** In a class, teacher wrote a question as given the differential equation $(x + y)\, dy + (x - y)\, dx = 0$ then he framed few questions and asked the students as

(*i*) the degree of differential equation is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) not defined

(*ii*) the given equation represents a/an

(*a*) equation, whose variables can be separated

(*b*) homogeneous equation

(*c*) linear differential equation of first order

(*d*) second order differential equation

(*iii*) the equation can be solved by substituting

(*a*) $y = vx$ (*b*) $y^2 = vx$ (*c*) $y = vx^2$ (*d*) $y = v^2x$

(*iv*) general solution of differential equation is

(*a*) $\tan^{-1}\left(\frac{y}{x}\right) + \frac{1}{2}\log|x^2 + y^2| = C$ (*b*) $\log(x + y) + \frac{1}{2}|x^2 + y^2| = C$

(*c*) $\tan^{-1}\left(\frac{y}{x}\right) + \tan^{-1}\left(\frac{y^2}{x^2}\right) = C$ (*d*) $\tan\left(\frac{y}{x}\right) + \frac{1}{2}\log|x^2 + y^2| = C$

($v$) particular solution, when $y = 1$, $x = 1$ is

($a$) $\tan^{-1}\left(\frac{y}{x}\right) + \frac{1}{2}\log(x^2 + y^2) = \frac{\pi}{4} + \frac{1}{2}\log 2$

($b$) $\log|x + y| + \frac{1}{2}\log|x^2 + y^2| = \frac{3}{2}\log 2$

($c$) $\tan^{-1}\left(\frac{y}{x}\right) + \tan^{-1}\left(\frac{y^2}{x^2}\right) = \frac{\pi}{2}$

($d$) $\tan\frac{y}{x} + \frac{1}{2}\log|x^2 + y^2| = \tan 1 + \frac{1}{2}\log 2$

**Sol.** Consider equation $(x + y)\, dy + (x - y)\, dx = 0$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{y - x}{x + y} \quad \ldots(i)$$

($i$) ($b$), degree of differential equation is 1.

($ii$) ($b$)

($iii$) ($a$)

($iv$) ($a$), Let $y = vx \quad \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

Substituting in $\frac{dy}{dx} = \frac{y - x}{x + y}$, we get

$$v + x\frac{dv}{dx} = \frac{vx - x}{x + vx} = \frac{v - 1}{1 + v}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v - 1}{1 + v} - v = \frac{v - 1 - v - v^2}{1 + v} = \frac{-(1 + v^2)}{1 + v}$$

$$\Rightarrow \quad \frac{1 + v}{1 + v^2}dv = -\frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{1 + v}{1 + v^2}\, dv = -\int \frac{dx}{x}$$

$$\Rightarrow \quad \int \frac{1}{1 + v^2}\, dv + \int \frac{v}{1 + v^2}\, dv = -\int \frac{dx}{x}$$

$$\Rightarrow \quad \tan^{-1} v + \frac{1}{2}\log|1 + v^2| = -\log|x| + C$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) + \frac{1}{2}\log\left|1 + \frac{y^2}{x^2}\right| = -\log|x| + C$$

$$\Rightarrow \quad \tan^{-1}\left(\frac{y}{x}\right) + \frac{1}{2}\log|x^2 + y^2| = C, \; C \text{ is constant of integration} \quad \ldots(ii)$$

(*v*) (*a*) Given $x = 1$, when $y = 1$

$\Rightarrow$ $$\tan^{-1} 1 + \frac{1}{2}\log 2 = C \Rightarrow C = \frac{\pi}{4} + \frac{1}{2}\log 2$$

From (*ii*), we get

$\tan^{-1}\left(\frac{y}{x}\right) + \frac{1}{2}\log|x^2 + y^2| = \frac{\pi}{4} + \frac{1}{2}\log 2$ is the particular solution of the given differential equation.

**37.** Friends are revising differential equations and working with the question $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$, few questions are required to be answered.

(*i*) The degree of the differential equation is

(*a*) 2 (*b*) 1 (*c*) 0 (*d*) not defined

(*ii*) The equation can be solved using

(*a*) method of separating variables

(*b*) method of homogeneous equation

(*c*) method of solving $\frac{dy}{dx} + P(x) \cdot y = Q(x)$

(*d*) method of solving $\frac{dx}{dy} + P(y) \cdot x = Q(y)$

(*iii*) $\int \frac{v-1}{v} dv$ is

(*a*) $\log\left|\frac{v-1}{v}\right| + C$ (*b*) $\log |v| + C$ (*c*) $1 - \log |v| + C$ (*d*) $v - \log|v| + C$

(*iv*) $\int \frac{dx}{x} = g(x) + C$, then $g(x)$ is

(*a*) $\frac{1}{2}$ (*b*) $-\frac{1}{x^2}$ (*c*) $e^x$ (*d*) $\log|x|$

(*v*) General solution of differential equation is

(*a*) $v - \log |vx| = C$ (*b*) $y + \log |y| = C$

(*c*) $y - x \log |y| = C$ (*d*) $y - x \log |y| = Cx$

**Sol.** Consider differential equation $\frac{dy}{dx} = \frac{y^2}{xy - x^2}$ ...(*i*)

(*i*) (*b*), degree is 1.

(*ii*) (*b*)

(*iii*) (*d*), $$\int \frac{v-1}{v} dv = \int\left(1 - \frac{1}{v}\right)dv = v - \log|v| + C$$

(*iv*) (*d*), $$\int \frac{1}{x} dx = \log|x| + C = g(x) + C$$

$\Rightarrow$ $$g(x) = \log|x|$$

(*v*) (*d*), Let $y = vx \Rightarrow \frac{dy}{dx} = v\cdot 1 + x\cdot\frac{dv}{dx}$

Substituting in (*i*), we get

$$v + x\cdot\frac{dv}{dx} = \frac{v^2x^2}{vx^2 - x^2} = \frac{v^2}{v-1}$$

$$\Rightarrow \quad x\frac{dv}{dx} = \frac{v^2}{v-1} - v$$

$$\Rightarrow \quad x\cdot\frac{dv}{dx} = \frac{v^2 - v^2 + v}{v-1} = \frac{v}{v-1}$$

$$\Rightarrow \quad \frac{v-1}{v}dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\int\frac{v-1}{v}dv = \int\frac{dx}{x}$$

$$\Rightarrow \quad \int\left\{1-\frac{1}{v}\right\}dv = \int\frac{dx}{x}$$

$$\Rightarrow \quad v - \log|v| = \log|x| + C$$

$$\Rightarrow \quad \frac{y}{x} - \log\left|\frac{y}{x}\right| - \log|x| = C$$

$$\Rightarrow \quad \frac{y}{x} - \log\left|\frac{y}{x}\cdot x\right| = C$$

$$\Rightarrow \quad \frac{y}{x} - \log|y| = C \text{ is required solution.}$$

**38.** Friends are trying to solve some queries related to the differential equations, one of the differential equation is $(1 + \sin^2 x)dy + (1 + y^2)\cos x\, dx = 0,\ y\left(\frac{\pi}{2}\right) = 0$

They are searching solution on the net, trying to get answers from friends on social sites, you also must have got the message, try to answer their questions.

(*i*) The order of differential equation is

(*a*) 2 (*b*) 0 (*c*) 1 (*d*) not defined

(*ii*) We can solve the equation using

(*a*) separating the variables method

(*b*) homogeneous equations method

(*c*) linear differential equation of first order method

(*d*) reducing the equation using substitution and representing in form (*b*) or (*c*)

(*iii*) $\int\frac{1}{1+y^2}dy$ is

(*a*) $\log|1 + y^2| + C$ (*b*) $\frac{\log|1+y^2|}{2y} + C$

(*c*) $\sin^{-1} y + C$ (*d*) $\tan^{-1} y + C$

(*iv*) Solution of differential equation is

(*a*) $\tan^{-1} y + \tan^{-1}(\sin x) = C$
(*b*) $\tan^{-1} y + \tan^{-1}(\cos x) = C$
(*c*) $\tan^{-1} y - \tan^{-1}(\sin x) = C$
(*d*) $\tan^{-1} y - \tan^{-1}(\cos x) = C$

(*v*) Given $y = 0$ for $x = \frac{\pi}{2}$ then solution is

(*a*) $\tan^{-1} y + \tan^{-1}(\sin x) = \frac{\pi}{4}$
(*b*) $\tan^{-1} y + \tan^{-1}(\cos x) = 0$
(*c*) $\tan^{-1} y - \tan^{-1}(\sin x) + \frac{\pi}{4} = 0$
(*d*) $y = \cos x$

**Sol.** (*i*) (*c*), (*ii*) (*a*), (*iii*) (*d*), (*iv*) (*a*), (*v*) (*a*)

**39.** Rate of change of one quantity with respect to other is represented as function $\frac{dy}{dx} = 2y + \cos 3x$, we need to find the general function and during the course of solution few questions we can answer

(*i*) the order of differential equation is

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) not defined

(*ii*) The given differential equation is of the form

(*a*) separating the variables
(*b*) homogeneous equations
(*c*) linear differential equation of type $\frac{dy}{dx} + P(x) \cdot y = Q(x)$
(*d*) linear differential equation of type $\frac{dx}{dy} + P(y) \cdot x = Q(y)$

(*iii*) Integrating factor for the solution of differential equation is

(*a*) $2x$ (*b*) $e^{2x}$ (*c*) $e^{-2x}$ (*d*) $e^{-2}$

(*iv*) $\int e^{-2x} \cos 3x \, dx$ is equal to

(*a*) $\frac{e^{-2x}}{13}(3 \sin 3x - 2 \cos 3x) + C$
(*b*) $\frac{e^{-2x}}{13}(3 \sin 3x + 2 \cos 3x) + C$
(*c*) $\frac{e^{-2x}}{13}(3 \cos 3x - 2 \sin 3x) + C$
(*d*) $\frac{e^{-2x}}{13}(3 \cos 3x + 2 \sin 3x) + C$

(*v*) Solution of differential equation is

(*a*) $y = \frac{1}{13}(3 \sin 3x - 2 \cos 3x) + C \cdot e^{2x}$
(*b*) $y = \frac{1}{13}(3 \sin 3x + 2 \cos 3x) + C \cdot e^{2x}$
(*c*) $y = \frac{1}{13}(3 \cos 3x - 2 \sin 3x) + C \cdot e^{2x}$
(*d*) $y = \frac{1}{13}(3 \cos 3x + 2 \sin 3x) + C \cdot e^{2x}$

**Sol.** (*i*) (*b*), Consider equation $\frac{dy}{dx} - 2y = \cos 3x$, order = 1

(*ii*) (*c*)

(*iii*) (*c*), Here $P(x) = -2$, $Q(x) = \cos 3x$

$\therefore$ Integrating factor (I.F.) $= e^{-\int 2dx} = e^{-2x}$

(*iv*) (*a*), Consider $I = \int \underset{\text{(I)}}{e^{-2x}} \underset{\text{(II)}}{\cos 3x} \, dx$

$$= e^{-2x} \cdot \frac{\sin 3x}{3} - \int (-2 \cdot e^{-2x}) \frac{\sin 3x}{3} \cdot dx$$

$$= \frac{1}{3}e^{-2x}\sin 3x + \frac{2}{3}\int \underset{\text{(I)}}{e^{-2x}} \underset{\text{(II)}}{\sin 3x}\, dx$$

$$= \frac{1}{3}e^{-2x}\sin 3x + \frac{2}{3}\left[e^{-2x}\left(\frac{-\cos 3x}{3}\right) - \int (-2\cdot e^{-2x})\left(\frac{-\cos 3x}{3}\right)dx\right]$$

$$I = \frac{1}{3}e^{-2x}\sin 3x - \frac{2}{9}e^{-2x}\cos 3x - \frac{4}{9}I$$

$$\Rightarrow \quad \frac{13}{9}I = \frac{e^{-2x}}{9}(3\sin 3x - 2\cos 3x]$$

$$\Rightarrow \quad I = \frac{e^{-2x}}{13}(3\sin 3x - 2\cos 3x)$$

(*v*) (*a*), solution is $\quad (\text{I.F.})\, y = \int \{(\text{I.F.})\, Q(x)\}\, dx$

$$\Rightarrow \quad e^{-2x}\cdot y = \int e^{-2x}\cdot \cos 3x\, dx$$

$$\Rightarrow \quad e^{-2x}\cdot y = \frac{e^{-2x}}{13}(3\sin 3x - 2\cos 3x) + C \qquad [\text{from } (iv)]$$

$$\Rightarrow \quad y = \frac{1}{13}(3\sin 3x - 2\cos 3x) + C\, e^{2x} \text{ is general solution.}$$

**40.** The differential equation related to a given situation is given by $\left[y - x\cos\left(\frac{y}{x}\right)\right]dy + \left[y\cos\left(\frac{y}{x}\right) - 2x\sin\left(\frac{y}{x}\right)\right]dx = 0$. Some of the conclusion drawn from given information can be taken as

(*i*) The order of differential equation is

(*a*) 1 (*b*) 0 (*c*) not defined (*d*) 2

(*ii*) The given equation is a

(*a*) trigonometric equation (*b*) homogeneous equation

(*c*) linear differential equation (*d*) linear differential equation of first order

(*iii*) The given equation can be solved using

(*a*) separating the variables

(*b*) by substituting $y = vx$

(*c*) by finding integrating factor

(*d*) by collecting all the trigonometric terms together

(*iv*) If $\int \frac{v - \cos v}{2\sin v - v^2}dv = f(v) + C$, then $f(v)$ is

(*a*) $2\sin v - v^2 + C$ (*b*) $-\frac{1}{2}\log|2\sin v - v^2| + C$

(*c*) $v - \cos v + C$ (*d*) $-\frac{1}{2}\log|v - \cos v| + C$

(*v*) Solution of differential equation is

(*a*) $2x^2 \sin y - xy^2 = C$ (*b*) $2x^2 \sin\left(\frac{y}{x}\right) + y^2 = C$

(*c*) $2x^2 \sin\left(\frac{y}{x}\right) - y = C$ (*d*) $2x^2 \sin\left(\frac{y}{x}\right) - y^2 = C$

**Sol.** (*i*) (*a*), Consider $\left[y - x\cos\left(\frac{y}{x}\right)\right]dy + \left[y\cos\left(\frac{y}{x}\right) - 2x\sin\left(\frac{y}{x}\right)\right]dx = 0$

$$\Rightarrow \quad \frac{dy}{dx} = \frac{2x\sin\left(\frac{y}{x}\right) - y\cos\left(\frac{y}{x}\right)}{y - x\cos\left(\frac{y}{x}\right)} \quad \text{order} = 1 \qquad \ldots(i)$$

(*ii*) (*b*)

(*iii*) (*b*)

(*iv*) (*b*), as $\int \frac{v - \cos v}{2\sin v - v^2} dv = -\frac{1}{2}\int \frac{1}{t} dt$ | Let $2\sin v - v^2 = t$ $\Rightarrow (2\cos v - 2v)dv = dt$

$$= -\frac{1}{2}\log|t| + C = -\frac{1}{2}\log|2\sin v - v^2| + C$$

(*v*) (*d*), Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

From (*i*), we get

$$v + x\frac{dv}{dx} = \frac{2x\sin v - vx\cos v}{vx - x\cos v} = \frac{2\sin v - v\cos v}{v - \cos v}$$

$$x\frac{dv}{dx} = \frac{2\sin v - v\cos v}{v - \cos v} - v$$

$$= \frac{2\sin v - v\cos v - v^2 + v\cos v}{v - \cos v} = \frac{2\sin v - v^2}{v - \cos v}$$

$$\Rightarrow \quad \frac{v - \cos v}{2\sin v - v^2} dv = \frac{dx}{x}$$

Integrating both sides, we get

$$\int \frac{v - \cos v}{2\sin v - v^2} dv = \int \frac{dx}{x}$$

$$\Rightarrow \quad -\frac{1}{2}\int \frac{1}{t} dt = \int \frac{dx}{x}$$ | Let $2\sin v - v^2 = t$ $\Rightarrow (2\cos v - 2v)dv = dt$

$$\Rightarrow \quad -\frac{1}{2}\log|t| = \log|x| + \log C, \ C \text{ is constant}$$

$$\Rightarrow \quad \log|t|^{-\frac{1}{2}} = \log|Cx| \Rightarrow \frac{1}{\sqrt{t}} = Cx$$

$$\Rightarrow \quad Cx\sqrt{2\sin v - v^2} = 1$$

$$\Rightarrow \quad Cx\sqrt{2\sin\left(\frac{y}{x}\right) - \frac{y^2}{x^2}} = 1$$

$$\Rightarrow \quad C\sqrt{2x^2\sin\left(\frac{y}{x}\right) - y^2} = 1$$

$\Rightarrow 2x^2\sin\left(\frac{y}{x}\right) - y^2 = k$, $k$ (constant) $= \frac{1}{C^2}$ (constant) is the required solution.

# 10 VECTOR ALGEBRA

## Multiple Choice Questions

**1.** Mathematically a vector is defined as a

(*a*) line segment (*b*) directed line segment

(*c*) line (*d*) ray

**Sol.** (*b*)

**2.** A vector equally inclined to axes is **[KVS]**

(*a*) $\hat{i}+\hat{j}+\hat{k}$ (*b*) $\hat{i}-\hat{j}+\hat{k}$ (*c*) $\hat{i}-\hat{j}-\hat{k}$ (*d*) $-\hat{i}+\hat{j}-\hat{k}$

**Sol.** (*a*), as direction ratios are 1, 1, 1 and direction cosines are $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

$\Rightarrow \cos\alpha = \cos\beta = \cos\gamma \Rightarrow \alpha = \beta = \gamma$

**3.** The position vector of a point which divides the join of points with position vectors $\vec{a}+\vec{b}$ and $2\vec{a}-\vec{b}$ in the ratio 1 : 2 internally is

(*a*) $\frac{3\vec{a}+2\vec{b}}{3}$ (*b*) $\vec{a}$ (*c*) $\frac{5\vec{a}-\vec{b}}{3}$ (*d*) $\frac{4\vec{a}+\vec{b}}{3}$

**Sol.** (*d*), as position vector $= \frac{2(\vec{a}+\vec{b})+1(2\vec{a}-\vec{b})}{1+2} = \frac{4\vec{a}+\vec{b}}{3}$

**4.** For which value of $p$, is $(\hat{i}+\hat{j}+\hat{k})p$ a unit vector?

(*a*) $\pm\frac{1}{\sqrt{3}}$ (*b*) $\pm\sqrt{3}$ (*c*) $\pm 1$ (*d*) $\pm\frac{1}{3}$

**Sol.** (*a*), as for a unit vector, $|p\hat{i}+p\hat{j}+p\hat{k}|=1$

$\Rightarrow \sqrt{p^2+p^2+p^2}=1 \quad \Rightarrow \quad p=\pm\frac{1}{\sqrt{3}}$

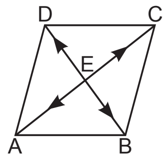
**5.** *ABCD* is a rhombus whose diagonals intersect at *E*. Then $\overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED}$ equals. **[CBSE 2020]**

(*a*) $\vec{0}$ (*b*) $\overrightarrow{AD}$ (*c*) $2\overrightarrow{BC}$ (*d*) $2\overrightarrow{AD}$

**Sol.** (*a*), as $\overrightarrow{EB} = -\overrightarrow{ED}$

and $\overrightarrow{EA} = -\overrightarrow{EC}$

$\Rightarrow \overrightarrow{EA}+\overrightarrow{EB}+\overrightarrow{EC}+\overrightarrow{ED} = \vec{0}$



**6.** Given vector $\overrightarrow{PQ} = 2\hat{i} + \hat{j} - 3\hat{k}$ and position vector of point $P$ is $3\hat{j} - 2\hat{k}$, then position vector of point $Q$ is

(*a*) $2\hat{i} - 2\hat{j} - \hat{k}$ (*b*) $2\hat{i} + 4\hat{j} - 5\hat{k}$ (*c*) $2\hat{i} + 4\hat{j} - \hat{k}$ (*d*) $2\hat{i} - 2\hat{j} - 5\hat{k}$

**Sol.** (*b*), as $\overrightarrow{PQ}$ = position vector of $Q$ – position vector of $P$

$$\Rightarrow \quad \text{Position vector of } Q = 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{j} - 2\hat{k}$$
$$= 2\hat{i} + 4\hat{j} - 5\hat{k}$$

**7.** A vector in the direction of vector $\hat{i} - 2\hat{j} + 2\hat{k}$ that has magnitude 15 is

(*a*) $\dfrac{\hat{i} - 2\hat{j} + 2\hat{k}}{3}$ (*b*) $15\hat{i} - 30\hat{j} + 30\hat{k}$

(*c*) $\hat{i} - 2\hat{j} + 15\hat{k}$ (*d*) $5\hat{i} - 10\hat{j} + 10\hat{k}$

**Sol.** (*d*), as vector $= 15\left(\dfrac{\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{1+4+4}}\right) = 5\hat{i} - 10\hat{j} + 10\hat{k}$

**8.** If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$ then the range of $|\lambda\vec{a}|$ is **[KVS]**

(*a*) [0, 8] (*b*) [–12, 8] (*c*) [0, 12] (*d*) [8, 12]

**Sol.** (*c*), as $|\lambda\vec{a}| = |\lambda||\vec{a}| = 4|\lambda|$

Also $-3 \le \lambda \le 2 \Rightarrow 0 \le |\lambda| \le 3$

$\Rightarrow 0 \le 4|\lambda| \le 12$

**9.** If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\,\vec{a} - \vec{b}$ to be a unit vector?

(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 90°

**Sol.** (*a*), as $|\sqrt{3}\,\vec{a} - \vec{b}|^2 = (\sqrt{3}\,\vec{a} - \vec{b})^2 = 3\vec{a}^2 + \vec{b}^2 - 2\sqrt{3}\,\vec{a}\cdot\vec{b}$

$$\Rightarrow \quad 1 = 3 + 1 - 2\sqrt{3}\,\vec{a}\cdot\vec{b}$$
$$\Rightarrow \quad \vec{a}\cdot\vec{b} = \frac{\sqrt{3}}{2}$$
$$\therefore \quad \cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\sqrt{3}}{2} = \cos 30°$$
$$\Rightarrow \quad \theta = 30°$$

**10.** The value of $\lambda$ for which vectors $2\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{i} - \lambda\hat{j} + 4\hat{k}$ are orthogonal is

(*a*) 12 (*b*) –12 (*c*) 14 (*d*) –14

**Sol.** (*c*), as $(2\hat{i} + \hat{j} + 3\hat{k})\cdot(\hat{i} - \lambda\hat{j} + 4\hat{k}) = 0$

$\Rightarrow 2 - \lambda + 12 = 0 \quad \Rightarrow \quad \lambda = 14$

**11.** The angle between the vectors $\vec{a} = \hat{i} - \hat{j} + \hat{k}$ and $\vec{b} = \hat{i} + \hat{j} - \hat{k}$ is

(*a*) $\frac{1}{3}$ (*b*) $\frac{2}{3}$ (*c*) $-\frac{1}{3}$ (*d*) $\cos^{-1}\left(-\frac{1}{3}\right)$

**Sol.** (*d*), as $\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{1-1-1}{\sqrt{3}\sqrt{3}} = -\frac{1}{3}$

$\Rightarrow \theta = \cos^{-1}\left(-\frac{1}{3}\right)$

**12.** If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is 60°, then $\vec{a}\cdot\vec{b}$ is

(*a*) $\sqrt{3}$ (*b*) 2 (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{\sqrt{3}}$

**Sol.** (*a*), as $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos 60° = \sqrt{3}\times 2\times\frac{1}{2} = \sqrt{3}$.

**13.** If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, then angle between $\vec{a}$ and $\vec{b}$ is

(*a*) 0° (*b*) 90° (*c*) 180° (*d*) 60°

**Sol.** (*b*), as $|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2 \Rightarrow (\vec{a}+\vec{b})^2 = (\vec{a}-\vec{b})^2$

$\Rightarrow \vec{a}\cdot\vec{b} = 0 \Rightarrow \theta = 90°$

**14.** If $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors, then the value of $|2\hat{a}+\hat{b}+\hat{c}|$ is

(*a*) $\sqrt{5}$ (*b*) $\sqrt{3}$ (*c*) $\sqrt{2}$ (*d*) $\sqrt{6}$

**Sol.** (*d*), as $\hat{a}, \hat{b}$ and $\hat{c}$ are mutually perpendicular unit vectors.

$\therefore \quad |\hat{a}| = |\hat{b}| = |\hat{c}| = 1$

and $\quad \hat{a}\cdot\hat{b} = \hat{b}\cdot\hat{c} = \hat{c}\cdot\hat{a} = 0 \quad$ ...(*i*)

Consider,
$$|2\hat{a}+\hat{b}+\hat{c}|^2 = (2\hat{a}+\hat{b}+\hat{c})^2$$
$$= 4\hat{a}^2 + \hat{b}^2 + \hat{c}^2 + 4\hat{a}\cdot\hat{b} + 4\hat{a}\cdot\hat{c} + 2\hat{b}\cdot\hat{c}$$
$$= 4|\hat{a}|^2 + |\hat{b}|^2 + |\hat{c}|^2 + 0 + 0 + 0 = 4 + 1 + 1 = 6$$

$\Rightarrow \quad |2\hat{a}+\hat{b}+\hat{c}| = \sqrt{6}$

**15.** The length of the sum of the three mutually perpendicular unit vectors is

(*a*) $\sqrt{5}$ (*b*) $\sqrt{3}$ (*c*) $\sqrt{2}$ (*d*) $\sqrt{6}$

**Sol.** (*b*), Let three mutually perpendicular unit vectors be $\vec{a}, \vec{b}$ and $\vec{c}$.

$\Rightarrow \quad |\vec{a}| = |\vec{b}| = |\vec{c}| = 1;$

$\vec{a}\cdot\vec{b} = 0,\ \vec{b}\cdot\vec{c} = 0,\ \vec{c}\cdot\vec{a} = 0 \quad$ ...(*i*)

Consider
$$|\vec{a}+\vec{b}+\vec{c}|^2 = (\vec{a}+\vec{b}+\vec{c})^2$$
$$= \vec{a}^2 + \vec{b}^2 + \vec{c}^2 + 2\vec{a}\cdot\vec{b} + 2\vec{b}\cdot\vec{c} + 2\vec{c}\cdot\vec{a}$$
$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = |\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 + 0 + 0 + 0$$
$$= 1 + 1 + 1$$
$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}|^2 = 3$$
$$\Rightarrow |\vec{a}+\vec{b}+\vec{c}| = \sqrt{3}$$

**16.** If for non zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b}$ is a unit vector and $|\vec{a}| = |\vec{b}| = \sqrt{2}$, then angle $\theta$ between vectors $\vec{a}$ and $\vec{b}$ is

(*a*) $\frac{\pi}{2}$ (*b*) $\frac{\pi}{3}$ (*c*) $\frac{\pi}{6}$ (*d*) $-\frac{\pi}{2}$

**Sol.** (*c*), as $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

**17.** The area of a parallelogram whose one diagonal is $2\hat{i} + \hat{j} - 2\hat{k}$ and one side is $3\hat{i} + \hat{j} - \hat{k}$ is

(*a*) $\hat{i} - 4\hat{j} - \hat{k}$ (*b*) $3\sqrt{2}$ sq units (*c*) $6\sqrt{2}$ sq units (*d*) 6 sq units

**Sol.** (*b*), as area of parallelogram $= \left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & 1 & -1 \end{matrix} \right\| = |\hat{i} - 4\hat{j} - \hat{k}| = \sqrt{1+16+1} = 3\sqrt{2}$ sq units

**18.** The angle between two vectors $\vec{a}$ and $\vec{b}$ with magnitudes 1 and 2 respectively and $|\vec{a} \times \vec{b}| = \sqrt{3}$ is

(*a*) 0° (*b*) $\frac{\pi}{3}$ (*c*) $\frac{\pi}{4}$ (*d*) $\frac{\pi}{6}$

**Sol.** (*b*), as $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{\sqrt{3}}{1 \times 2} \Rightarrow \sin\theta = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$

**19.** If $(2\hat{i} + 6\hat{j} + 14\hat{k}) \times (\hat{i} - \lambda\hat{j} + 7\hat{k}) = \vec{0}$ then $\lambda$ is

(*a*) $\frac{27}{2}$ (*b*) $-\frac{27}{2}$ (*c*) 3 (*d*) –3

**Sol.** (*d*), as $\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 6 & 14 \\ 1 & -\lambda & 7 \end{vmatrix} = \vec{0} \Rightarrow \hat{i}(42 + 14\lambda) - \hat{j}(14 - 14) + \hat{k}(-2\lambda - 6) = \vec{0} \Rightarrow \lambda = -3$

**20.** The value of $p$ for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors is

(*a*) 3 (*b*) $\frac{3}{2}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{1}{3}$

**Sol.** (*c*), as $\frac{3}{1} = \frac{2}{p} = \frac{9}{3} \Rightarrow p = \frac{2}{3}$

**21.** If $|\vec{a}| = 8$, $|\vec{b}| = 3$ and $|\vec{a}\cdot\vec{b}| = 12\sqrt{3}$ then the value of $|\vec{a} \times \vec{b}|$ is

(*a*) 12 (*b*) $12\sqrt{3}$ (*c*) 6 (*d*) $4\sqrt{3}$

**Sol.** (*a*), as $|\vec{a} \times \vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2$

$$\Rightarrow \quad |\vec{a} \times \vec{b}|^2 = 64 \times 9 - 144 \times 3 = 576 - 432 = 144$$

$$\Rightarrow \quad |\vec{a} \times \vec{b}| = 12$$

**22.** Vectors $\vec{a}$ and $\vec{b}$ are such that $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = \frac{2}{3}$ and $(\vec{a} \times \vec{b})$ is a unit vector. The angle between $\vec{a}$ and $\vec{b}$ is

(*a*) $\frac{\pi}{3}$ (*b*) $\frac{\pi}{4}$ (*c*) $\frac{\pi}{6}$ (*d*) $\frac{\pi}{2}$

**Sol.** (*a*), as $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{1 \times 3}{\sqrt{3} \times 2} = \frac{\sqrt{3}}{2} \Rightarrow \theta = \frac{\pi}{3}$

**23.** Position vectors of points $A$ and $B$ are $\vec{a} + \vec{b}$ and $2\vec{a} - \vec{b}$. Then $\vec{AB}$ equal to

(*a*) $3\vec{a}$ (*b*) $-\vec{a} + 2\vec{b}$ (*c*) $\vec{a} - 2\vec{b}$ (*d*) none of these

**Sol.** (*c*), as $\overrightarrow{AB}$ = Position vector of $B$ – Position vector of $A$

**24.** Given vector $\vec{a}$, then $-2\vec{a}$ is a vector whose

(*a*) magnitude is twice that of $\vec{a}$ and direction is same as that of $\vec{a}$

(*b*) magnitude is twice that of $\vec{a}$ and direction is opposite to that of $\vec{a}$

(*c*) magnitude is same as that of $\vec{a}$ and direction is opposite to that of $\vec{a}$

(*d*) none of these

**Sol.** (*b*), result related to $\vec{a}$ and $k\vec{a}$, $k$ is scalar.

**25.** If $\vec{a}$ and $\vec{b}$ are non-zero vectors, such that $\vec{a} \cdot \vec{b} = 0$, then

(*a*) $\vec{a}$ is parallel to $\vec{b}$ (*b*) $\vec{a}$ and $\vec{b}$ are collinear

(*c*) $\vec{a}$ is perpendicular to $\vec{b}$ (*d*) none of these

**Sol.** (*c*), result

**26.** If $\vec{a} + \vec{b} + \vec{c} = \vec{0}$ then

(*a*) $\vec{a} \times \vec{b} = \vec{b} \times \vec{c} = \vec{c} \times \vec{a}$ (*b*) $\vec{a} + \vec{b} = \vec{b} + \vec{c} = \vec{c} + \vec{a}$

(*c*) $\vec{a}, \vec{b}, \vec{c}$ are non-coplanar (*d*) none of these

**Sol.** (*a*), as $\vec{a} + \vec{b} + \vec{c} = \vec{0} \Rightarrow \vec{a} \times \vec{a} + \vec{a} \times \vec{b} + \vec{a} \times \vec{c} = \vec{0}$

$\Rightarrow \quad \vec{a} \times \vec{b} = \vec{c} \times \vec{a}$

Similarly $\quad \vec{b} \times \vec{c} = \vec{a} \times \vec{b}$

**27.** Area of parallelogram, whose diagonals are along vectors $\hat{i} + 2\hat{k}$ and $2\hat{j} - 3\hat{k}$ is

(*a*) $\sqrt{29}$ (*b*) $-4\hat{i} + 3\hat{j} + 2\hat{k}$ (*c*) $\frac{1}{2}\sqrt{29}$ (*d*) none of these

**Sol.** (*c*), as area $= \frac{1}{2}|(\hat{i} + 2\hat{k}) \times (2\hat{j} - 3\hat{k})|$

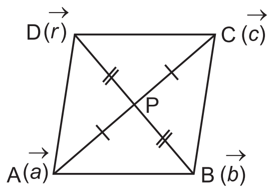
**28.** If $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of vertices $A$, $B$, $C$ of a parallelogram $ABCD$, then the position vector of $D$ is

(*a*) $\vec{a}+\vec{c}-\vec{b}$ (*b*) $\vec{a}-\vec{c}+\vec{b}$ (*c*) $\vec{a}-\vec{c}-\vec{b}$ (*d*) $\vec{c}-\vec{a}+\vec{b}$

**Sol.** (*a*), as in a parallelogram, diagonals bisect each other.

Using this concept, we get $\dfrac{\vec{a}+\vec{c}}{2} = \dfrac{\vec{b}+\vec{r}}{2}$

$\Rightarrow \quad \vec{r} = \vec{a} + \vec{c} - \vec{b}$




**29.** The position vector of point $R$ which divides the line segment joining the points $P$ and $Q$ whose position vectors are $(2\vec{a}+\vec{b})$ and $(\vec{a}-3\vec{b})$ externally in the ratio 1 : 2 is

(*a*) $3\vec{a}+5\vec{b}$ (*b*) $\dfrac{3\vec{a}+5\vec{b}}{3}$ (*c*) $3\vec{a}+\vec{b}$ (*d*) $\dfrac{3\vec{a}+\vec{b}}{3}$

**Sol.** (*a*), as $\vec{r} = \dfrac{2(2\vec{a}+\vec{b})-(\vec{a}-3\vec{b})}{2-1} = 3\vec{a}+5\vec{b}$

**30.** If $\vec{a} \times \vec{b} = \hat{i}+\hat{j}+\hat{k}$ and $|\vec{a}| = 2$, $|\vec{b}| = 1$, then the angle between $\vec{a}$ and $\vec{b}$ is

(*a*) 30° (*b*) 60° (*c*) 90° (*d*) 120°

**Sol.** (*b*), as $|\vec{a}\times\vec{b}| = |\hat{i}+\hat{j}+\hat{k}| = \sqrt{1+1+1} = \sqrt{3}$

$$\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}|\,|\vec{b}|} = \frac{\sqrt{3}}{2\times 1} = \frac{\sqrt{3}}{2} = \sin 60° \Rightarrow \theta = 60°$$

**31.** The position vector of point $R$ which divides the line segment joining two points $P$ and $Q$ with position vectors $2\vec{a}+\vec{b}$ and $\vec{a}-2\vec{b}$ in the ratio 1 : 3 internally is

(*a*) $7\vec{a}+\vec{b}$ (*b*) $7\vec{a}-\vec{b}$ (*c*) $\dfrac{7\vec{a}+\vec{b}}{4}$ (*d*) $\dfrac{\vec{a}+7\vec{b}}{4}$

**Sol.** (*c*), as position vector of $R = \dfrac{3(2\vec{a}+\vec{b})+1(\vec{a}-2\vec{b})}{1+3} = \dfrac{7\vec{a}+\vec{b}}{4}$

**32.** The position vector of a point through which the line $\vec{r} = 2\hat{i}-\hat{j}+4\hat{k}+\lambda(\hat{i}-\hat{j}-\hat{k})$ passes is

(*a*) <2, –1, 4> (*b*) $\hat{i}-\hat{j}-\hat{k}$ (*c*) $2\hat{i}-\hat{j}+4\hat{k}$ (*d*) (3, –2, 3)

**Sol.** (*c*), as equation of line is $\vec{r} = \vec{a}+\lambda\vec{b}$, $\vec{a}$ represents position vector of a point through which line passes.

Hence, the position vector of the point $= 2\hat{i}-\hat{j}+4\hat{k}$.

**33.** If $\vec{a}$ and $\vec{b}$ are unit vectors, then $\vec{a}\cdot\vec{b}$ can be

(*a*) $\sqrt{3}$ (*b*) $\sqrt{2}$ (*c*) $\sqrt{6}$ (*d*) $\frac{1}{2}\sqrt{2}$

**Sol.** (*d*), as

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

$$|\vec{a}| = |\vec{b}| = 1 \qquad [\because \vec{a} \text{ and } \vec{b} \text{ are unit vectors}]$$

$\Rightarrow$ $\vec{a}\cdot\vec{b} = \cos\theta$, as $\cos\theta \not> 1$. Hence, (*d*) is true.

**34.** If $|\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = 144$ and $|\vec{a}| = 8$, then $|b|$ is

(*a*) 3 (*b*) 2 (*c*) $\frac{3}{2}$ (*d*) 1

**Sol.** (*c*), as $|\vec{a}\times\vec{b}|^2 + (\vec{a}\cdot\vec{b})^2 = |\vec{a}|^2|\vec{b}|^2\sin^2\theta + |\vec{a}|^2|\vec{b}|^2\cos^2\theta$

$$= |\vec{a}|^2|\vec{b}|^2(\sin^2\theta + \cos^2\theta) = |\vec{a}|^2|\vec{b}|^2.$$

$\therefore$ $144 = |\vec{a}|^2|\vec{b}|^2 \Rightarrow 144 = (8)^2\,|\vec{b}|^2 \Rightarrow |\vec{b}|^2 = \frac{144}{64} = \frac{9}{4}$

$\Rightarrow$ $|\vec{b}| = \frac{3}{2}$.

**35.** If $\vec{x}$ is a unit vector such that $\vec{x}\times\hat{i} = \hat{k}$, then $\vec{x}\cdot\hat{j}$ is

(*a*) –1 (*b*) 1 (*c*) 0 (*d*) not defined

**Sol.** (*a*), as $-\hat{j}\times i = \hat{k} \Rightarrow \vec{x}\,.\,\hat{j} = -\hat{j}\,.\,\hat{j} = -1$

**36.** For any two vectors $\vec{a}$ and $\vec{b}$, if $|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2\,|\vec{b}|^2 - k$, then $k$ is

(*a*) $(\vec{a}\,\vec{b})^2$ (*b*) $(\vec{a}\cdot\vec{b})^2$ (*c*) $\vec{a}\,\vec{b}$ (*d*) $\vec{a}\cdot\vec{b}$

**Sol.** (*b*), as we have $|\vec{a}\times\vec{b}|^2 = |\vec{a}|^2|\vec{b}|^2 - (a\cdot b)^2 \Rightarrow k = (\vec{a}\cdot\vec{b})^2$

**37.** The area of a parallelogram whose diagonals are $2\hat{i}$ and $-3\hat{k}$ is

(*a*) 6 sq units (*b*) 2 sq units (*c*) 12 sq units (*d*) 3 sq units

**Sol.** (*d*), as

$$\text{area} = \frac{1}{2}|(2\hat{i})\times(-3\hat{k})|$$

$$= 3|\hat{j}| = 3\times 1 = 3 \text{ sq units}$$

**38.** The value of $p$, for which $p(\hat{i}+\hat{j}+\hat{k})$ is a unit vector is

(*a*) 1 (*b*) –1 (*c*) $\pm\frac{1}{3}$ (*d*) $\pm\frac{1}{\sqrt{3}}$

**Sol.** (*d*), as $|p(\hat{i}+\hat{j}+\hat{k})| = 1$

$\Rightarrow$ $|p||\hat{i}+\hat{j}+\hat{k}| = 1$

$\Rightarrow$ $|p|\sqrt{1+1+1} = 1 \Rightarrow p = \pm\frac{1}{\sqrt{3}}$

**39.** The value of $\lambda$ for which the vectors $2\hat{i} - \lambda\hat{j} + \hat{k}$ and $\hat{i} + 2\hat{j} - \hat{k}$ are orthogonal is

(*a*) 2 (*b*) $\frac{1}{2}$ (*c*) $-1$ (*d*) 1

**Sol.** (*b*), as if vectors are orthogonal, then

$$2 \times 1 + (-\lambda) \times 2 + (1)(-1) = 0$$

$$\Rightarrow \quad 2 - 2\lambda - 1 = 0 \Rightarrow \lambda = \frac{1}{2}$$

**40.** If $|\vec{a}| = 2$, $|\vec{b}| = 3$ and $\vec{a} \cdot \vec{b} = 4$, then the value of $|\vec{a} + 2\vec{b}|$ is

(*a*) $2\sqrt{14}$ (*b*) 4 (*c*) 3 (*d*) $3\sqrt{14}$

**Sol.** (*a*), as

$$\begin{aligned} |\vec{a} + 2\vec{b}|^2 &= (\vec{a} + 2\vec{b})^2 \\ &= \overrightarrow{a^2} + 4\overrightarrow{b^2} + 4\vec{a} \cdot \vec{b} \\ &= |\vec{a}|^2 + 4|\vec{b}|^2 + 4\vec{a} \cdot \vec{b} \\ &= (2)^2 + 4(3)^2 + 4 \times 4 \\ &= 4 + 36 + 16 = 56 \end{aligned}$$

$$\therefore \quad |\vec{a} + 2\vec{b}| = \sqrt{56} = 2\sqrt{14}$$

**41.** Unit vectors along vector $\hat{i} + 2\hat{j} - 2\hat{k}$ are

(*a*) $\pm(\hat{i} + 2\hat{j} - 2\hat{k})$ (*b*) $\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}$

(*c*) $\pm\left(\frac{1}{3}\hat{i} + \frac{2}{3}\hat{j} - \frac{2}{3}\hat{k}\right)$ (*d*) none of these

**Sol.** (*c*)

**42.** The position vectors of opposite vertices of a parallelogram are $2\vec{a} + 3\vec{b}$ and $\vec{a} - 2\vec{b}$. Then position vector of the point of intersection of diagonals is

(*a*) $3\vec{a} + \vec{b}$ (*b*) $\frac{\vec{a} + 5\vec{b}}{2}$ (*c*) $\frac{3\vec{a} + \vec{b}}{2}$ (*d*) none of these

**Sol.** (*c*)

**43.** If for non-zero vectors $\vec{a}, \vec{b}, \vec{c}$, $\vec{a} \times \vec{b} = \vec{c}$ and $\vec{b} \times \vec{c} = \vec{a}$, then

(*a*) $\vec{a}, \vec{b}$ and $\vec{c}$ are parallel to each other

(*b*) $\vec{a}, \vec{b}, \vec{c}$ are perpendicular to each other

(*c*) $\vec{c} \times \vec{a} = \vec{b} \times \vec{a}$

(*d*) none of these

**Sol.** (*b*)

**44.** If $|\vec{a}+\vec{b}|=|\vec{a}-\vec{b}|$, then

(*a*) $\vec{a}$ is parallel to $\vec{b}$ (*b*) $\vec{a}$ is perpendicular to $\vec{b}$

(*c*) $|\vec{a}|=|\vec{b}|$ (*d*) $\vec{a}=\vec{b}$

**Sol.** (*b*)

**45.** If $|\vec{a}|$ = 5, $|\vec{b}|$ = 13 and $|\vec{a}\times\vec{b}|$ = 25, then $\vec{a}\cdot\vec{b}$ is equal to

(*a*) 12 (*b*) 5 (*c*) 13 (*d*) 60 **[KVS]**

**Sol.** (*d*)

## Assertion-Reason Questions

(*a*) If both A and R are true and R is the correct explanation of A.

(*b*) If both A and R are true but R is not the correct explanation of A.

(*c*) A is true, R is false.

(*d*) A is false, R is true.

**46.** **Assertion (A):** The vectors $\vec{a}=2\hat{i}-\hat{j}+5\hat{k}$ and $\vec{b}=5\hat{i}+\hat{j}-3\hat{k}$ are perpendicular to each other.

**Reason (R):** $\vec{a}\times\vec{b}$ is a vector perpendicular to both $\vec{a}$ and $\vec{b}$.

**Sol.** (*d*), as

$$\vec{a}\cdot\vec{b} = 10-1-15=-6\neq 0$$

If two vectors are perpendicular then their dot product is 0. Hence $\vec{a}$ and $\vec{b}$ are not perpendicular. A is false

R is true, as $\vec{a}\times\vec{b}$ is vector

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}$$

$\hat{n}$ is a unit vector perpendicular to both $\vec{a}$ and $\vec{b}$.

**47.** **Assertion (A):** The points $A(-2\hat{i}+3\hat{j}+5\hat{k}), B(\hat{i}+2\hat{j}+3\hat{k})$ and $C(7\hat{i}-\hat{k})$ are collinear

**Reason (R):** A, B, and C are collinear if $|\overrightarrow{AB}|+|\overrightarrow{BC}|=|\overrightarrow{AC}|$

**Sol.** (*a*), since points are $A(-2\hat{i}+3\hat{j}+5\hat{k}), B(\hat{i}+2\hat{j}+3\hat{k})$ and $C(7\hat{i}-\hat{k})$.

$$\overrightarrow{AB} = 3\hat{i}-\hat{j}-2\hat{k}$$

$$|\overrightarrow{AB}| = \sqrt{9+1+4}=\sqrt{14}$$

$$\overrightarrow{BC} = 6\hat{i}-2\hat{j}-4\hat{k}$$

$$|\overrightarrow{BC}| = \sqrt{36+4+16}=\sqrt{56}=2\sqrt{14}$$

$$\overrightarrow{AC} = 9\hat{i}-3\hat{j}-6\hat{k}$$

$$|\overrightarrow{AC}| = \sqrt{81+9+36}=\sqrt{126}=3\sqrt{14}$$

$$|\overrightarrow{AB}|+|\overrightarrow{BC}| = \overrightarrow{AC}$$

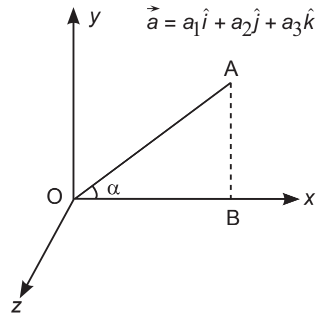
So, points are collinear. A is true, R is also true and the correct explanation of A.

**48.** **Assertion (A):** The projection of vector $\hat{i}+3\hat{j}+7\hat{k}$ on the vector $7\hat{i}-\hat{j}+8\hat{k}$ is $\frac{60}{\sqrt{114}}$.

**Reason (R):** If $\alpha, \beta$ and $y$ are the angles made by vector $a = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ with coordinate axis, then

$$l = \frac{a_1}{|\vec{a}|}, \quad m = \frac{a_2}{|\vec{a}|}, \quad n = \frac{a_3}{|\vec{a}|}$$

**Sol.** (*b*), projection of vector $\vec{a}$ on $\vec{b}$ $= \frac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$




So, projection of $\hat{i}+3\hat{j}+7\hat{k}$ on $7\hat{i}-\hat{j}+8\hat{k}$ is

$$= \frac{7-3+56}{\sqrt{49+1+64}} = \frac{60}{\sqrt{114}}$$

A is true

$$l = \cos\alpha = \frac{|\overrightarrow{OB}|}{|\overrightarrow{AO}|} = \frac{a_1}{|\vec{a}|}$$

Similarly $m = \cos\beta = \frac{a_2}{|\vec{a}|}, \quad n = \cos\gamma = \frac{a_3}{|\vec{a}|}$

R is also true, but not the correct explanation of A.

**49.** **Assertion (A):** If $\vec{a} = \hat{i}+3\hat{j}+4\hat{k}$ and $\vec{b} = -\hat{i}+3\hat{j}+5\hat{k}$ represent the two adjacent sides of a parallelogram the area of parallelogram is $3\sqrt{14}$ square units.

**Reason (R):** $(\vec{a}-\vec{b})\times(\vec{a}+\vec{b}) = |\vec{a}|^2 - |\vec{b}|^2$

**Sol.** (*c*), as

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 3 & 4 \\ -1 & 3 & 5 \end{vmatrix} = 3\hat{i}-9\hat{j}+6\hat{k}$$

$$|\vec{a}\times\vec{b}| = \sqrt{9+81+36} = \sqrt{126} = 3\sqrt{14}$$

Area of parallelogram $= |\vec{a}\times\vec{b}| = 3\sqrt{14}$ square units

A is true.

$$(\vec{a}-\vec{b})\times(\vec{a}+\vec{b}) = \vec{a}\times\vec{a}+\vec{a}\times\vec{b}-\vec{b}\times\vec{a}-\vec{b}\times\vec{b}$$
$$= 0+\vec{a}\times\vec{b}+\vec{a}\times\vec{b}-0 = 2(\vec{a}\times\vec{b})$$

R is false.

## Case-based Questions

**50.** A man is watching an aeroplane which is at the coordinate point $A(4, -1, 3)$ assuming that the man is at $O(0, 0, 0)$. At the sametime he saw a bird at the coordinate point $B(2, 0, 4)$. Based on the above information answer the following:

(*i*) The representation of position vector $\overrightarrow{AB}$ is

(*a*) $2\hat{i}+\hat{j}+\hat{k}$ (*b*) $-2\hat{i}+\hat{j}+\hat{k}$ (*c*) $\hat{i}+2\hat{j}+3\hat{k}$ (*d*) $4\hat{i}+\hat{j}+4\hat{k}$

(*ii*) The distance between aeroplane and bird is

(*a*) 6 units (*b*) $\sqrt{8}$ units (*c*) $\sqrt{6}$ units (*d*) $2\sqrt{6}$ units

(*iii*) The unit vector along $\overrightarrow{AB}$ is

(*a*) $\frac{2}{6}\hat{i}+\frac{1}{6}\hat{j}+\frac{1}{6}\hat{k}$ (*b*) $\frac{-2}{\sqrt{6}}\hat{i}+\frac{1}{6}\hat{j}+\frac{1}{6}\hat{k}$

(*c*) $\frac{-2}{\sqrt{6}}\hat{i}+\frac{1}{\sqrt{6}}\hat{j}+\frac{1}{\sqrt{6}}\hat{k}$ (*d*) $\frac{4}{\sqrt{6}}\hat{i}+\frac{2}{\sqrt{6}}\hat{j}+\frac{3}{\sqrt{6}}\hat{k}$

(*iv*) The direction cosines of $\overrightarrow{AB}$ are

(*a*) $<\frac{-2}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}}>$ (*b*) $<\frac{-2}{\sqrt{6}},\frac{1}{6},\frac{1}{6}>$

(*c*) $<\frac{4}{\sqrt{6}},\frac{2}{\sqrt{6}},\frac{3}{\sqrt{6}}>$ (*d*) $<-2, 1, 1>$

(*v*) The angles which $\overrightarrow{AB}$ makes with $x$, $y$ and $z$ axes are

(*a*) $\cos^{-1}\left(\frac{2}{\sqrt{6}}\right), \cos^{-1}\left(\frac{1}{\sqrt{6}}\right), \cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$ (*b*) $\cos^{-1}\left(\frac{-2}{\sqrt{6}}\right), \cos^{-1}\left(\frac{1}{\sqrt{6}}\right), \cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$

(*c*) $\cos^{-1}\left(\frac{4}{\sqrt{6}}\right), \cos^{-1}\left(\frac{2}{\sqrt{6}}\right), \cos^{-1}\left(\frac{3}{\sqrt{6}}\right)$ (*d*) $\cos^{-1}(-2), \cos^{-1}(1), \cos^{-1}(1)$

**Sol.** (*i*) (*b*), $\overrightarrow{AB} = (2-4)\hat{i} + (0+1)\hat{j} + (4-3)\hat{k} = -2\hat{i} + \hat{j} + \hat{k}$

(*ii*) (*c*), $|\overrightarrow{AB}| = \sqrt{(-2)^2+(1)^2+(1)^2}$

$= \sqrt{6}$ units

(*iii*) (*c*), Unit vector along $AB = \frac{\overrightarrow{AB}}{|\overrightarrow{AB}|} = \frac{-2\hat{i}+\hat{j}+\hat{k}}{\sqrt{6}} = \frac{-2\hat{i}}{\sqrt{6}}+\frac{\hat{j}}{\sqrt{6}}+\frac{\hat{k}}{\sqrt{6}}$

(*iv*) (*a*), Direction cosines of $\overrightarrow{AB}$ are $<\frac{-2}{\sqrt{6}},\frac{1}{\sqrt{6}},\frac{1}{\sqrt{6}}>$

(*v*) (*b*),

$$\cos\alpha = \frac{-2}{\sqrt{6}} \Rightarrow \alpha = \cos^{-1}\left(\frac{-2}{6}\right)$$

$$\cos\beta = \frac{1}{\sqrt{6}} \Rightarrow \beta = \cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$$

$$\cos\gamma = \frac{1}{\sqrt{6}} \Rightarrow \gamma = \cos^{-1}\left(\frac{1}{\sqrt{6}}\right)$$

**51.** Employee in a office are following social distance and during lunch they are sitting at places marked by points $A$, $B$ and $C$. Each one is representing position as $A(\hat{i}-2\hat{j}+4\hat{k})$, $B(5\hat{i}+2\hat{k})$ and $C(3\hat{i}+2\hat{j}+4\hat{k})$.

Based on the above information answer the following:

(*i*) The distance between $A$ and $B$ is

(*a*) $4\sqrt{6}$ units (*b*) $2\sqrt{6}$ units (*c*) $3\sqrt{6}$ units (*d*) $6\sqrt{2}$ units (*ii*) The distance between $B$ and $C$ is

(*a*) $4\sqrt{6}$ units (*b*) $2\sqrt{3}$ units (*c*) $4\sqrt{3}$ units (*d*) $3\sqrt{3}$ units

(*iii*) The position vector $\overrightarrow{AB}$ is

(*a*) $4\hat{i} + 2\hat{k}$ (*b*) $4\hat{i} + 3\hat{k} + 2\hat{j}$ (*c*) $4\hat{i} + 2\hat{j} - 2\hat{k}$ (*d*) $3\hat{i} + 2\hat{j} - 2\hat{k}$

(*iv*) The unit vector along $\overrightarrow{AB}$ is

(*a*) $\frac{2}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$ (*b*) $\frac{2}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$

(*c*) $\frac{-2}{\sqrt{6}}\hat{i} + \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$ (*d*) $\frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} + \frac{1}{\sqrt{6}}\hat{k}$

(*v*) The area enclosed by $A$, $B$ and $C$ is

(*a*) $2\sqrt{41}$ sq units (*b*) $2\sqrt{14}$ sq units

(*c*) $2\sqrt{8}$ sq units (*d*) $3\sqrt{6}$ sq units

**Sol.** (*i*) (*b*), (*ii*) (*b*), (*iii*) (*c*), (*iv*) (*b*), (*v*) (*b*)

**52.** A class XII student appearing for a competitive examination was asked to attempt the following questions.

Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be three non zero vectors. **[CBSE Question Bank]**

(*i*) If $\vec{a}$ and $\vec{b}$ are such that $|\vec{a} + \vec{b}| = |\vec{a} - \vec{b}|$then

(*a*) $\vec{a} \perp \vec{b}$ (*b*) $\vec{a} \parallel \vec{b}$ (*c*) $\vec{a} = \vec{b}$ (*d*) none of these

(*ii*) If $\vec{a} = \hat{i} - 2\hat{j}$, $\vec{b} = 2\hat{i} + \hat{j} + 3\hat{k}$ then $(2\vec{a} + \vec{b}) \cdot [(\vec{a} + \vec{b}) \times (\vec{a} - 2\vec{b})]$ is

(*a*) 0 (*b*) 4 (*c*) 3 (*d*) 2

(*iii*) If $\vec{a}$ and $\vec{b}$ are unit vectors and $\theta$ be the angle between them then $|\vec{a} - \vec{b}|$ is

(*a*) $\sin\frac{\theta}{2}$ (*b*) $2\sin\frac{\theta}{2}$ (*c*) $2\cos\frac{\theta}{2}$ (*d*) $\cos\frac{\theta}{2}$

(*iv*) Let $\vec{a}$, $\vec{b}$ and $\vec{c}$ be unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$ then $\vec{a}$ is

(*a*) $2(\vec{b} \times \vec{c})$ (*b*) $-2(\vec{b} \times \vec{c})$ (*c*) $\pm 2(\vec{b} \times \vec{c})$ (*d*) $2(\vec{b} \pm \vec{c})$

(*v*) The area of the parallelogram formed by $\vec{a}$ and $\vec{b}$ as diagonals is[use $\vec{a}$ and $\vec{b}$ from (*ii*)]

(*a*) 70 (*b*) 35 (*c*) $\frac{\sqrt{70}}{2}$ (*d*) $\sqrt{70}$

**Sol.** (*i*) (*a*), (*ii*) (*a*), (*iii*) (*b*), (*iv*) (*c*), (*v*) (*c*)

# 11 THREE DIMENSIONAL GEOMETRY

## Multiple Choice Questions

**1.** The distance of point (2, 5, 7) from the *x*-axis is

(*a*) 2 (*b*) $\sqrt{74}$ (*c*) $\sqrt{29}$ (*d*) $\sqrt{53}$

**Sol.** (*b*), as distance of point (2, 5, 7) from the *x*-axis is $\sqrt{5^2+7^2} = \sqrt{25+49} = \sqrt{74}$

**2.** *P* is a point on the line segment joining the points (3, 5, –1) and (6, 3, –2). If *y*-coordinate of point *P* is 2, then its *x*-coordinate will be

(*a*) 2 (*b*) $\frac{17}{3}$ (*c*) $\frac{15}{2}$ (*d*) –5

**Sol.** (*c*), as let *P* divides the join of (3, 5, –1) and (6, 3, –2) in the ratio *k* : 1

$$\therefore \quad \frac{3k+5}{k+1} = 2$$

$$\Rightarrow \quad 3k + 5 = 2k + 2 \Rightarrow k = -3$$

$\therefore$ *x*-coordinate is

$$\frac{6k+3}{k+1} = \frac{-18+3}{-3+1} = \frac{15}{2}$$

**3.** Direction ratios of a line are 2, 3, –6. Then direction cosines of a line making obtuse angle with the *y*-axis are

(*a*) $\frac{2}{7}, \frac{-3}{7}, \frac{-6}{7}$ (*b*) $\frac{-2}{7}, \frac{3}{7}, \frac{-6}{7}$ (*c*) $\frac{-2}{7}, \frac{-3}{7}, \frac{6}{7}$ (*d*) $\frac{-2}{7}, \frac{-3}{7}, \frac{-6}{7}$

**Sol.** (*c*), as direction cosines of a line whose direction ratio are 2, 3, –6 are $\frac{2}{7}, \frac{3}{7}, \frac{-6}{7}$. As angle with the *y*-axis is obtuse.

$\therefore$ $\cos\beta < 0$,

Therefore direction ratios are $\frac{-2}{7}, \frac{-3}{7}, \frac{6}{7}$.

**4.** A line makes angle α, β, γ with *x*-axis, *y*-axis and *z*-axis respectively then $\cos 2\alpha + \cos 2\beta + \cos 2\gamma$ is equal to **[KVS]**

(*a*) 2 (*b*) 1 (*c*) –2 (*d*) –1

**Sol.** (*d*), as $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$$\Rightarrow \frac{1+\cos 2\alpha}{2} + \frac{1+\cos 2\beta}{2} + \frac{1+\cos 2\gamma}{2} = 1$$

$$\Rightarrow \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$$

**5.** The line joining the points (0, 5, 4) and (1, 3, 6) meets *XY*-plane at the point

(*a*) (–2, 9, 0) (*b*) (4, –3, 0) (*c*) (1, –2, 0) (*d*) (1, 3, 0)

**Sol.** (*a*), as line is $\frac{x-1}{1} = \frac{y-3}{-2} = \frac{z-6}{2} = \lambda$ (say)

General point on line is $(\lambda + 1, -2\lambda + 3, 2\lambda + 6)$

If it meets *XY*-plane, then $2\lambda + 6 = 0 \Rightarrow \lambda = -3$

$\therefore$ Point is (–2, 9, 0)

**6.** If the direction cosines of a given line are $\frac{1}{k}, \frac{1}{k}, \frac{1}{k}$ then, the value of $k$ is

(*a*) $\frac{1}{\sqrt{2}}$ (*b*) $\pm\frac{1}{\sqrt{3}}$ (*c*) 1 (*d*) $\pm\sqrt{3}$

**Sol.** (*d*) as $\frac{1}{k}, \frac{1}{k}, \frac{1}{k}$ are direction cosines of a line

$$\therefore \quad \frac{1}{k^2} + \frac{1}{k^2} + \frac{1}{k^2} = 1 \Rightarrow \frac{3}{k^2} = 1$$

$$\Rightarrow \quad k^2 = 3$$

$$\Rightarrow \quad k = \pm\sqrt{3}$$

**7.** The equations of $y$-axis in space are

(*a*) $x = 0, y = 0$ (*b*) $x = 0, z = 0$ (*c*) $y = 0, z = 0$ (*d*) $y = 0$

**Sol.** (*b*), as on the $y$-axis, $x$-coordinate and $z$-coordinate are zeroes.

**8.** A line makes angles $\frac{\pi}{4}, \frac{3\pi}{4}$ with $x$-axis and $y$-axis respectively. Then the angle which it makes with $z$-axis can be

(*a*) $\frac{\pi}{2}$ (*b*) $\frac{\pi}{6}$ (*c*) $\frac{\pi}{3}$ (*d*) 0 or $\pi$

**Sol.** (*d*), as $\cos^2\frac{\pi}{4} + \cos^2\frac{3\pi}{4} + \cos^2\gamma = 1$

$$\Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2\gamma = 1 \Rightarrow \cos\gamma = 0 \Rightarrow \gamma = 0, \pi$$

**9.** If the direction cosines of a line are $\frac{k}{3}, \frac{k}{3}, \frac{k}{3}$, then value of $k$ is

(*a*) $k > 0$ (*b*) $0 < k < 1$ (*c*) $k = \frac{1}{3}$ (*d*) $k = \pm\sqrt{3}$

**Sol.** (*d*), as $3 \times \frac{k^2}{9} = 1 \Rightarrow k = \pm\sqrt{3}$

**10.** If $\alpha, \beta, \gamma$ are the angles that a line makes with $x$, $y$ and $z$-axes respectively then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is

(*a*) 1 (*b*) –1 (*c*) 2 (*d*) –2

**Sol.** (*c*), as $\alpha, \beta, \gamma$ are the angles which a line makes with *x*, *y* and *z* – axes

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

**11.** Equation of the perpendicular drawn from the point with position vector $2\hat{i} - \hat{j} + \hat{k}$ to the plane $\vec{r} \cdot (\hat{i} - 3\hat{k}) = 5$ is

(*a*) $\vec{r} = \lambda(\hat{i} - 3\hat{k})$
(*b*) $\vec{r} = \lambda(2\hat{i} - \hat{j} + \hat{k})$
(*c*) $\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(\hat{i} - 3\hat{k})$
(*d*) $\vec{r} = (\hat{i} - 3\hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

**Sol.** (*c*), as $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and direction of line is along $\vec{n} = \hat{i} - 3\hat{k}$.

**12.** Distance of plane $\vec{r} \cdot (2\hat{i} + 3\hat{j} - 6\hat{k}) + 2 = 0$, from origin is

(*a*) 2
(*b*) 14
(*c*) $\frac{2}{7}$
(*d*) $-\frac{2}{7}$

**Sol.** (*c*), as normal form of plane is

$$\vec{r} \cdot \left(-\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = \frac{2}{7}$$

$$\therefore \qquad p = \frac{2}{7}$$

$\therefore$ distance $= \frac{2}{7}$

**13.** The distance of the point (2, 3, 4) from the plane $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) = -11$ is

(*a*) 1 unit
(*b*) $\frac{1}{7}$ units
(*c*) $\frac{11}{7}$ units
(*d*) 2 units

**Sol.** (*a*), as distance of the point (2, 3, 4) from the plane $\vec{r} \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) + 11 = 0$ is

$$\left|\frac{(2\hat{i} + 3\hat{j} + 4\hat{k}) \cdot (3\hat{i} - 6\hat{j} + 2\hat{k}) + 11}{\sqrt{9 + 36 + 4}}\right| = \left|\frac{6 - 18 + 8 + 11}{7}\right| = 1 \text{ unit}$$

**14.** The vector normal to the plane $\vec{r} \cdot (3\hat{i} - 7\hat{k}) + 5 = 0$ is

(*a*) $\vec{r}$
(*b*) $3\hat{i} - 7\hat{j}$
(*c*) $3\hat{i} - 7\hat{k}$
(*d*) $3\hat{i} - 7\hat{k} + 5$

**Sol.** (*c*), as plane is $\vec{r} \cdot (3\hat{i} - 7\hat{k}) + 5 = 0$.

Normal vector is $\vec{n} = 3\hat{i} - 7\hat{k}$.

**15.** Distance between planes $\vec{r} \cdot (2\hat{i} + \hat{j} - 2\hat{k}) + 5 = 0$ and $\vec{r} \cdot (6\hat{i} + 3\hat{j} - 6\hat{k}) + 2 = 0$ is

(*a*) $\frac{9}{13}$
(*b*) $\frac{15}{4}$
(*c*) $\frac{13}{9}$
(*d*) $\frac{1}{13}$

**Sol.** (*c*), as distance $= \left|\frac{5 - \frac{2}{3}}{\sqrt{4 + 1 + 4}}\right| = \frac{13}{9}$ units

**16.** Intercept cut by the plane $2x - y + 2z + 7 = 0$ on the *x*-axis is

(*a*) 2
(*b*) $\frac{7}{2}$
(*c*) $-\frac{7}{2}$
(*d*) –2

**Sol.** (*c*), as plane is $2x - y + 2z = -7$

$$\Rightarrow \frac{x}{-\frac{7}{2}} + \frac{y}{7} + \frac{z}{-\frac{7}{2}} = 1$$

**17.** Direction ratios of a line passing through the points (2, 1, 0) and (3, 2, –1) are

(*a*) (1, 1, –1) (*b*) 1, 1, –1 (*c*) < 5, 3, –1 > (*d*) none of these

**Sol.** (*b*), as DR's are 3 – 2, 2 – 1, – 1 – 0, i.e. 1, 1, –1

**18.** Equation of the line passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$ is

(*a*) $\frac{x-1}{2} = \frac{y-2}{-7} = \frac{z-3}{4}$ (*b*) $\frac{x}{-2} = \frac{y}{7} = \frac{z}{-4}$

(*c*) $\frac{x-2}{-2} = \frac{y-1}{7} = \frac{z-3}{-4}$ (*d*) none of these

**Sol.** (*d*), as line is $\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c}$

and $a + 3b + 3c = 0$

$-3a + 2b + 5c = 0$

find $a$, $b$, $c$

**19.** The ratio in which the line segment joining the points (2, 4, 5) and (3, 5, – 4) is divided by *YZ*-plane is

(*a*) 5 : 4 internally (*b*) 4 : 5 externally

(*c*) 2 : 3 externally (*d*) none of these

**Sol.** (*c*), if ratio is $k : 1$, then

$$\frac{3k+2}{k+1} = 0 \Rightarrow k = -\frac{2}{3} \Rightarrow 2 : 3 \text{ externally.}$$

**20.** Direction cosines of a unit vector perpendicular to the plane $\vec{r} \cdot (6\hat{i} - 3\hat{j} - 2\hat{k}) - 1 = 0$ are

(*a*) 6, –3, –2 (*b*) $\frac{6}{7}, -\frac{3}{7}, -\frac{2}{7}$ (*c*) $-\frac{6}{7}, \frac{3}{7}, -\frac{2}{7}$ (*d*) none of these

**Sol.** (*b*), as plane is $\vec{r} \cdot (6\hat{i} - 3\hat{j} - 2\hat{k}) = 1$

$$\Rightarrow \vec{r} \cdot \left(\frac{6}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{2}{7}\hat{k}\right) = \frac{1}{7}$$

**21.** A line makes equal angles with axes, direction cosines of line are

(*a*) 1, 1, 1 (*b*) $\frac{1}{3}, \frac{1}{3}, \frac{1}{3}$ (*c*) $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$ (*d*) $\frac{1}{\sqrt{3}}, \frac{-1}{\sqrt{3}}, \frac{1}{\sqrt{3}}$

**Ans.** (*c*)

**22.** Projection of a line segment joining the points (2, 0, 5) and (0, 3, 1) on the line whose direction ratios are 2, 3, 6 is

(*a*) $-\frac{19}{7}$ (*b*) $\frac{19}{49}$ (*c*) $\frac{19}{7}$ (*d*) 19

**Ans.** (*c*)

**23.** Direction ratios of the line $\frac{4-x}{2} = \frac{y}{6} = \frac{1-z}{3}$ are

(*a*) 2, 6, 3 (*b*) –2, 6, 3 (*c*) 2, – 6, 3 (*d*) none of these

**Ans.** (*c*)

**24.** Distance between parallel planes $2x - y + 3z - 4 = 0$ and $6x - 3y + 9z + 13 = 0$ is

(*a*) 17 (*b*) $\frac{25}{3}$ (*c*) $\frac{25}{3\sqrt{14}}$ (*d*) $\frac{25}{\sqrt{14}}$

**Ans.** (*c*)

**25.** If line $\frac{x-1}{2} = \frac{y+3}{1} = \frac{z-5}{-1}$ is parallel to the plane $px + 3y - z + 5 = 0$, then the value of '$p$' is

(*a*) 2 (*b*) –2 (*c*) $\frac{1}{2}$ (*d*) none of these

**Ans.** (*b*)

**26.** The direction cosines of a line passing through the points (0, 1, 2) and (4, 4, 8) are

(*a*) $\frac{4}{\sqrt{61}}, \frac{3}{\sqrt{61}}, \frac{6}{\sqrt{61}}$ (*b*) $\frac{4}{61}, \frac{3}{61}, \frac{6}{61}$ (*c*) $\frac{4}{\sqrt{61}}, \frac{4}{\sqrt{61}}, \frac{8}{\sqrt{61}}$ (*d*) $\frac{4}{61}, \frac{4}{61}, \frac{8}{61}$

**Sol.** (*a*), as DR's are 4 – 0, 4 – 1, 8 – 2, i.e. 4, 3, 6

$$\text{Dc's: } \frac{4}{\sqrt{16+9+36}}, \frac{3}{\sqrt{16+9+36}}, \frac{6}{\sqrt{16+9+36}} = \frac{4}{\sqrt{61}}, \frac{3}{\sqrt{61}}, \frac{6}{\sqrt{61}}$$

**27.** A plane $2x - 3y + 6z - 11 = 0$ makes an angle $\sin^{-1} \alpha$ with the $x$-axis, the value of $\alpha$ is

(*a*) $\frac{4}{7}$ (*b*) $\frac{3}{7}$ (*c*) $\frac{5}{7}$ (*d*) $\frac{2}{7}$

**Sol.** (*d*), as $\sin(\sin^{-1} \alpha) = \frac{2.1 - 0 + 0}{\sqrt{1+0+0}\,\sqrt{4+9+36}} \Rightarrow \alpha = \frac{2}{7}$

**28.** The direction cosines of line of support of vector $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ are

(*a*) <1, –2, 2> (*b*) $<\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}>$

(*c*) $<\frac{1}{\sqrt{3}}, \frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}}>$ (*d*) <–1, 2, 2>

**Sol.** (*b*), as $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k} \Rightarrow \hat{a} = \frac{1}{3}\hat{i} - \frac{2}{3}\hat{j} + \frac{2}{3}\hat{k}$, therefore, DC's : $\frac{1}{3}, \frac{-2}{3}, \frac{2}{3}$

**29.** A line makes angles of 60° with $y$-axis and $z$-axis respectively, the angle which it makes with $x$-axis anticlockwise is

(*a*) 60° (*b*) 45° (*c*) –60° (*d*) –45°

**Sol.** (*b*), as $\cos^2 \alpha + \cos^2 60° + \cos^2 60° = 1$

$$\Rightarrow \cos^2\alpha + \frac{1}{4} + \frac{1}{4} = 1 \Rightarrow \cos^2 \alpha = \frac{1}{2}$$

$$\Rightarrow \cos \alpha = \frac{1}{\sqrt{2}} \Rightarrow \alpha = 45° \text{ (anticlockwise)}$$

**30.** If a line passes through the points $A(0, 3, 2)$ and $B(4, -1, 6)$, then the direction ratios of $AB$ are

(*a*) $<\frac{1}{3}, \frac{1}{3}, \frac{1}{3}>$ (*b*) $<\frac{1}{3}, -\frac{1}{3}, \frac{1}{3}>$ (*c*) $<1, -1, 1>$ (*d*) $<1, 1, 1>$

**Sol.** (*c*), As direction ratios of $AB$ are $4 - 0, -1 - 3, 6 - 2$, i.e. $4, -4, 4$

or $1, -1, 1$ i.e. $< 1, -1, 1 >$

**31.** The general point on the line $\vec{r} = (2\hat{i} + \hat{j} - 4\hat{k}) + \lambda(3\hat{i} + 2\hat{j} - \hat{k})$ is

(*a*) $(2, 1, -4)$ (*b*) $(3, 2, -1)$

(*c*) $(-1, 1, 3)$ (*d*) $(2 + 3\lambda, 1 + 2\lambda, -4 - \lambda)$

**Sol.** (*d*), As given line is $\vec{r} = \left(2\hat{i} + \hat{j} - 4\hat{k}\right) + \lambda\left(3\hat{i} + 2j - \hat{k}\right)$

$\therefore$ position vector of a point through which line passes.

$$\therefore \quad \vec{r} = (2 + 3\lambda)\hat{i} + (1 + 2\lambda)\hat{j} + (-4 - \lambda)\hat{k}$$

$\therefore$ General point is $(2 + 3\lambda, 1 + 2\lambda, -4 - \lambda)$

**32.** General equation of a plane passing through the point $(4, -1, 3)$ is

(*a*) $4x - y + 3z = 0$ (*b*) $a(x - 4) + b(y + 1) + c(z - 3) = 0$

(*c*) $4x - y + 3z = 26$ (*d*) $4x - y + 3z = 6$

**Sol.** (*b*), as general equation of the plane is $a(x - 4) + b(y + 1) + c(z - 3) = 0$, where $a, b, c$ are direction ratios of normal to the plane.

**33.** A line makes angles 45° and 60° with $x$-axis and $y$-axis respectively. The angle which it makes with the $z$-axis is

(*a*) $\frac{\pi}{3}$ (*b*) $\frac{\pi}{3}$ or $\frac{2\pi}{3}$ (*c*) $\frac{2\pi}{3}$ (*d*) $\frac{\pi}{6}$ or $\frac{5\pi}{6}$

**Sol.** (*b*), as $\cos^2 45° + \cos^2 60° + \cos^2\gamma = 1 \Rightarrow \frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1 \Rightarrow \cos^2\gamma = \frac{1}{4}$

$$\Rightarrow \quad \cos\gamma = \pm\frac{1}{2} \Rightarrow \gamma = \frac{\pi}{3} \text{ or } \frac{2\pi}{3}$$

**34.** The value of $\lambda$ for which the lines $\frac{1-x}{3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{6-z}{7}$ are perpendicular is

(*a*) $\frac{5}{2}$ (*b*) $\frac{-5}{2}$ (*c*) 2 (*d*) $-2$

**Sol.** (*d*), as lines are $\frac{x-1}{-3} = \frac{y-2}{2\lambda} = \frac{z-3}{2}$ and $\frac{x-1}{3\lambda} = \frac{y-1}{1} = \frac{z-6}{-7}$

If lines are perpendicular then $-9\lambda + 2\lambda - 14 = 0 \Rightarrow 7\lambda = -14 \Rightarrow \lambda = -2$

## Assertion-Reason Questions

**Directions: In the following questions, a statement of assertion (A) is followed by a statement of reason (R). Mark the correct choice as:**

(*a*) If both A and R are true and R is the correct explanation of A.

(*b*) If both A and R are true but R is not the correct explanation of A.

(*c*) A is true, R is false.

(*d*) A is false, R is true.

**35. Assertion (A):** The lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are intersecting.

**Reason (R):** If the shortest distance between two non parallel lines is zero then they are intersecting.

**Sol.** (*a*), for equation of first line

$$\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$$

Passing point, $\vec{a_1} = -3\hat{i} + \hat{j} + 5\hat{k}$

parallel vector $\vec{b_1} = -3\hat{i} + \hat{j} + 5\hat{k}$

For equation of second line $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$

Passing point, $\vec{a_2} = -\hat{i} + 2\hat{j} + 5\hat{k}$

Parallel vector $\vec{b}2 = -\hat{i} + 2\hat{j} + 5\hat{k}$

$$\vec{b_1} \times \vec{b}2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix}$$

$$= \hat{i}(5-10) - \hat{j}(-15+5) + \hat{k}(-6+1) = -5\hat{i} + 10\hat{j} - 5\hat{k}$$

$$\vec{a_2} - \vec{a_1} = (-\hat{i} + 2\hat{j} + 5\hat{k}) - (-3\hat{i} + \hat{j} + 5\hat{k}) = 2\hat{i} + \hat{j}$$

$$(\vec{a_2} - \vec{a_1}) \cdot (\vec{b_1} \times \vec{b_2}) = -10 + 10 = 0$$

So, lines are intersecting. 'A' is true.

Now the shortest distance between two lines $= \frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|}$

If shortest distance = 0

$$\frac{(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2)}{|\vec{b}_1 \times \vec{b}_2|} = 0$$

$$(\vec{a}_2 - \vec{a}_1) \cdot (\vec{b}_1 \times \vec{b}_2) = 0$$

$\Rightarrow$ lines are intersecting

R is also true and the correct explanation of A.

**36. Assertion (A):** The lines $\frac{x-1}{3}=\frac{y-1}{2}=\frac{z-2}{3}$ lies in a plane $2x - 9y + 4z = 1$.

**Reason (R):** If line is parallel to a plane then it will lie in the plane.

**Sol.** (*c*)

$$\frac{x-1}{3} = \frac{y-1}{2} = \frac{z-2}{3}$$

Point in line (1, 1, 2) vector parallel to line $\vec{b} = 3\hat{i} + 2\hat{j} + 3\hat{k}$

Equation of plane is $2x - 9y + 4z = 1$

$$\vec{N} = 2\hat{i} - 9\hat{j} + 4\hat{k}$$
$$\vec{b} \cdot \vec{N} = 6 - 18 + 12 = 0$$

So line is parallel to plane.

Now check weather point (1, 1, 2) satisfy the plane

$$2 - 9 + 8 = 1$$
$$\Rightarrow \quad 10 - 9 = 1$$
$$\Rightarrow \quad 1 = 1$$

Hence, line lies in the plane, A is true.

**37. Assertion (A):** The distance of a point from (–1, 0, 2) from plane $2x + 3y - 2z = 4$ is $\sqrt{5}$ units.

**Reason (R):** The distance of a plane from origin is $d$ if equation of plane is $\vec{r} \cdot \hat{n} = d$ where $\hat{n}$ is the unit vector perpendicular to plane.

**Sol.** (*d*),

The perpendicular distance of point (–1, 0, 2) from the plane $2x + 3y - 2z - 4 = 0$ is

$$d = \left| \frac{-2+0-4-4}{\sqrt{1+0+4}} \right| = \frac{10}{\sqrt{5}} \text{ units}$$
$$= \frac{10\sqrt{5}}{\sqrt{5}\times\sqrt{5}} = \frac{10\sqrt{5}}{5} = 2\sqrt{5} \text{ units}$$

A is false and R is true.

**38. Assertion (A):** The direction cosines of line $\frac{2x+1}{4}=\frac{y-1}{3}=\frac{3z+4}{6}$ is $\frac{2}{\sqrt{17}}, \frac{3}{\sqrt{17}}, \frac{4}{\sqrt{17}}$.

**Reason (R):** The distance between two parallel lines $\vec{r} = \vec{a}_1 + \lambda\vec{b}$ and $\vec{r} = \vec{a}_2 + \lambda\vec{b}$ is given by

$$d = \frac{\left|(\vec{a}_2 - \vec{a}_1)\times\vec{b}\right|}{|\vec{b}|}$$

**Sol.** (*b*)

The equation of line is $\quad \frac{2x+1}{4} = \frac{y-1}{3} = \frac{3z+4}{6}$

or $\quad \frac{x+\frac{1}{2}}{2} = \frac{y-1}{3} = \frac{z+\frac{4}{3}}{2}$

The direction ratios of line are 2, 3, 2

Direction cosines of the line are $\dfrac{2}{\sqrt{4+9+4}}, \dfrac{3}{\sqrt{4+9+4}}, \dfrac{2}{\sqrt{4+9+4}}$

i.e $\dfrac{2}{\sqrt{17}}, \dfrac{3}{17}, \dfrac{2}{17}$

A is true.

R is also true but not the correct explanation of A.

## Case-based Questions

**39.** A cricket match is organized between two clubs $A$ and $B$ for which a team from each club is chosen. Remaining players of Club $A$ and Club $B$ are respectively sitting on the plane represented by the equation $\vec{r} \cdot \left(2\vec{i} - \vec{j} + \vec{k}\right) = 3$ and $\vec{r} \cdot (\hat{i} + 3\hat{j} + 2\hat{k}) = 8$, to cheer the team of their own clubs.

**[CBSE Question Bank]**


Based on the above answer the following:

(*i*) The Cartesian equation of the plane on which players of club A are seated is

(*a*) $2x - y + z = 3$ (*b*) $2x - y + 2z = 3$

(*c*) $2x - y + z = -3$ (*d*) $x - y + z = 3$

(*ii*) The magnitude of the normal to the plane on which players of club B are seated, is

(*a*) $\sqrt{15}$ (*b*) $\sqrt{14}$ (*c*) $\sqrt{17}$ (*d*) $\sqrt{20}$

(*iii*) The intercept form of the equation of the plane on which players of club B are seated is

(*a*) $\dfrac{x}{8} + \dfrac{y}{\frac{8}{3}} + \dfrac{z}{2} = 1$ (*b*) $\dfrac{x}{5} + \dfrac{y}{\frac{8}{3}} + \dfrac{z}{3} = 1$

(*c*) $\dfrac{x}{8} + \dfrac{y}{\frac{8}{3}} + \dfrac{z}{4} = 1$ (*d*) $\dfrac{x}{8} + \dfrac{y}{7} + \dfrac{z}{2} = 1$

(*iv*) Which of the following is a player of club B?

(*a*) Player sitting at (1, 2, 1) (*b*) Player sitting at (0, 1, 2)

(*c*) Player sitting at (1, 4, 1) (*d*) Player sitting at (1, 1, 2)

(*v*) The distance of the plane, on which players of club B are seated, from the origin is

(*a*) $\frac{8}{\sqrt{14}}$ units (*b*) $\frac{6}{\sqrt{14}}$ units (*c*) $\frac{7}{\sqrt{14}}$ units (*d*) $\frac{9}{\sqrt{14}}$ units

**Sol.** (*i*) (*a*), $2x - y + z = 3$ (*ii*) (*b*), $\sqrt{14}$ (*iii*) (*c*), $\frac{x}{8} + \frac{y}{\frac{8}{3}} + \frac{z}{4} = 1$

(*iv*) (*d*), Player sitting at (1, 1, 2) (*v*) (*a*), $\frac{8}{\sqrt{14}}$ units

**40.** The Indian coast guard, while patrolling, saw a suspicious boat with people. They were nowhere looking like fishermen. The coast guard were closely observing the movement of the boat for an opportunity to seize the boat. They observed that the boat is moving along a planar surface. At an instant of time, the coordinates of the position of the coast guard helicopter and the boat is (1, 3, 5) and (2, 5, 3) respectively. **[CBSE Question Bank]**

Based on the above answer the following:

(*i*) If the line joining the positions of the helicopter and the boat is perpendicular to the plane in which the boat moves, then the equation of the plane is

(*a*) $-x + 2y - 2z = 6$ (*b*) $x + 2y + 2z = 6$

(*c*) $x + 2y - 2z = 6$ (*d*) $x - 2y - 2z = 6$

(*ii*) If the coast guard decide to shoot the boat at that given instant of time, then what is the distance (in metres) that the bullet has to travel?

(*a*) 5 m (*b*) 3 m (*c*) 6 m (*d*) 4 m

(*iii*) If the coast guard decides to shoot the boat at that given instant of time, when the speed of bullet is 36m/s, then what is the time taken for the bullet to travel and hit the boat?

(*a*) $\frac{1}{8}$ seconds (*b*) $\frac{1}{14}$ seconds (*c*) $\frac{1}{10}$ seconds (*d*) $\frac{1}{12}$ seconds

(*iv*) At that given instant of time, the equation of line passing through the positions of the helicopter and boat is

(*a*) $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z-5}{-2}$ (*b*) $\frac{x-1}{2}=\frac{y+3}{1}=\frac{z-5}{-2}$

(*c*) $\frac{x+1}{-2}=\frac{y-3}{-1}=\frac{z-5}{-2}$ (*d*) $\frac{x-1}{2}=\frac{y+3}{-1}=\frac{z+5}{2}$

(*v*) At a different instant of time, the boat moves to a different position along the planar surface. What should be the coordinates of the location of the boat if the coast guard shoots the bullet along the line whose equation is $\frac{x}{1}=\frac{y-1}{2}=\frac{z-2}{1}$ for the bullet to hit the boat?

(*a*) $\left(\frac{-8}{3},\frac{19}{3},\frac{-14}{3}\right)$ (*b*) $\left(\frac{8}{3},\frac{-19}{3},\frac{-14}{3}\right)$

(*c*) $\left(\frac{8}{3},\frac{-19}{3},\frac{14}{3}\right)$ (*d*) none of the above

**Sol.** (*i*) (*c*), $x + 2y - 2z = 6$ (*ii*) (*b*), 3m

(*iii*) (*d*), $\frac{1}{12}$ seconds (*iv*) (*a*), $\frac{x-1}{1}=\frac{y-3}{2}=\frac{z-5}{-2}$

(*v*) (*d*), none of the above.

**41.** Two non-parallel and non-intersecting straight lines are called skew lines. For skew lines, the line segment of the shortest distance will be perpendicular to both the lines. If the lines are $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$. Then, the shortest distance is given as

$$d = \left|\frac{(\vec{b_1}\times\vec{b_2})\cdot(\vec{a_2}-\vec{a_1})}{|\vec{b_1}\times\vec{b_2}|}\right|$$

Here, $\vec{a_1}, \vec{a_2}$ are position vectors of point through which the lines are passing and $\vec{b_1}, \vec{b_2}$ are the vectors in the direction of a line.

(*a*) If a line has the direction ratios –18, 12, –4 then what are its direction cosines?

(*i*) $\frac{-7}{11},\frac{6}{11},\frac{-3}{11}$ (*ii*) $\frac{8}{11},\frac{-2}{11},\frac{6}{11}$ (*iii*) $\frac{-9}{11},\frac{6}{11},\frac{-2}{11}$ (*iv*) $\frac{9}{11},\frac{-6}{11},\frac{2}{11}$

(*b*) Two lines $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$ are not coplanar if

(*i*) $(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times(\overrightarrow{-b_2})) = 0$ (*ii*) $(\vec{a_2}-\vec{a_1})\cdot(\vec{b_1}\times\vec{b_2}) = 0$

(*iii*) $(\vec{a_2}+\vec{a_1})\cdot(-\vec{b_1}\times\vec{b_2}) = 0$ (*iv*) $(\vec{a_2}-\vec{a_1})\cdot(-\vec{b_1}\times-\vec{b_2}) = 0$

(*c*) Write the distance of a point $P(a, b, c)$ from the $x$-axis

(*i*) $\sqrt{c^2 + a^2}$ units (*ii*) $\sqrt{b^2 + c^2 + a^2}$ units

(*iii*) $\sqrt{b^2 + c^2}$ units (*iv*) 0 units

(*d*) If the Cartesian equation of a line is $\frac{3-x}{5} = \frac{y+4}{7} = \frac{2z-6}{4}$, then write the vector equation of the line.

(*i*) $\vec{r} = (3\hat{i} - 4\hat{j} + 3\hat{k}) + \lambda(-5\hat{i} + 7\hat{j} + 2\hat{k})$ (*ii*) $\vec{r} = (4\hat{i} - 3\hat{j} + 4\hat{k}) + \lambda(-7\hat{i} + 5\hat{j} + 3\hat{k})$

(*iii*) $\vec{r} = (4\hat{i} - 3\hat{j} + 3\hat{k}) + \lambda(-5\hat{i} + 3\hat{j} + 3\hat{k})$ (*iv*) $\vec{r} = (4\hat{i} - 3\hat{j} + 5\hat{k}) + \lambda(-7\hat{i} + 5\hat{j} + 7\hat{k})$

(*e*) If a unit vector $\hat{a}$ makes angles $\frac{\pi}{3}$ with $\hat{i}$, $\frac{\pi}{4}$ with $\hat{j}$ and an acute angle $\theta$ with $\hat{k}$, then the value of $\theta$ is

(*i*) $\frac{\pi}{4}$ (*ii*) $\frac{\pi}{6}$ (*iii*) $\pi$ (*iv*) $\frac{\pi}{3}$

**Sol.** (*i*) (*c*), (*ii*) (*c*), (*iii*) (*c*), (*iv*) (*a*), (*v*) (*d*)

# 13 PROBABILITY

## Multiple Choice Questions

**1.** Let $A$ and $B$ be two given mutually exclusive events. Then $P(A/B)$ is

(*a*) 1 (*b*) $\frac{P(A \cup B)}{P(B)}$ (*c*) $\frac{P(A \cup B)}{P(B)}$ (*d*) 0

**Sol.** (*d*), as $P(A \cap B) = 0$ and $P(A/B) = \frac{P(A \cap B)}{P(B)}$.

**2.** Let $A$ and $B$ be two given events such that

$P(A) = 0.6$, $P(B) = 0.2$ and $P(A/B) = 0.5$. Then $P(A'/B')$ is **[KVS]**

(*a*) $\frac{1}{10}$ (*b*) $\frac{3}{10}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{6}{7}$

**Sol.** (*c*), as $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow P(A \cap B) = 0.5 \times 0.2 = 0.1$

$$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{3}{8}$$

**3.** If $A$ and $B$ are two independent events such that $P(A) = \frac{1}{7}$ and $P(B) = \frac{1}{6}$, then $P(A' \cap B')$ is

(*a*) $\frac{5}{7}$ (*b*) $\frac{6}{7}$ (*c*) $\frac{5}{6}$ (*d*) $\frac{1}{6}$

**Sol.** (*a*), as $P(A' \cap B') = P(A').\ P(B') = \left(1 - \frac{1}{7}\right)\left(1 - \frac{1}{6}\right)$

$= \frac{6}{7} \times \frac{5}{6} = \frac{5}{7}$

**4.** Given $P(A) = 0.2$, $P(B) = 0.3$ and $P(A \cap B) = 0.1$. then $P(A/B)$ is

(*a*) 0.2 (*b*) 0.3 (*c*) $\frac{P(A \cup B)}{P(B)}$ (*d*) $\frac{1}{3}$

**Sol.** (*a*), as $P(A/B) = \frac{P(A \cap B)}{P(A)} = \frac{0.1}{0.3} = \frac{1}{3}$

**5.** Given $P(A) = 0.4$, $P(B) = 0.7$ and $P(B/A) = 0.6$ then $P(A \cup B)$ is

(*a*) 1.1 (*b*) 0.86 (*c*) 0.46 (*d*) 0.16

**Sol.** (*b*), as $P(B/A) = \frac{P(A \cap B)}{P(A)}$

$\Rightarrow 0.6 \times 0.4 = P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = 0.24$

Now, $\quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.4 + 0.7 - 0.24 = 0.86$

**6.** Three persons $A$, $B$ and $C$, fire a target in turn. Their probabilities of hitting the target are 0.2, 0.3 and 0.5 respectively, the probability that target hit is

(*a*) 0.993 (*b*) 0.94 (*c*) 0.72 (*d*) 0.90

**Sol.** (*c*), as $P$(target hit) = $P$(at least one hits the target)

$= 1 - P(\text{none hits})$

$= 1 - P(\bar{A}\ \bar{B}\ \bar{C})$

**7.** Let $A$ and $B$ be two given independent events such that $P(A) = p$ and $P(B) = q$ and $P$(exactly one of $A$, $B$) $= \frac{2}{3}$, then value of $3p + 3q - 6pq$ is

(*a*) 2 (*b*) –2 (*c*) 4 (*d*) –4

**Sol.** (*a*), as $P(A)\,P(\bar{B}) + P(\bar{A})\,P(B) = \frac{2}{3}$

$\Rightarrow p \cdot (1-q) + (1-p)q = \frac{2}{3}$

$\Rightarrow p - pq + q - pq = \frac{2}{3} \Rightarrow 3p + 3q - 6pq = 2$

**8.** If $P(A \cap B) = 70\%$ and $P(B) = 85\%$, then $P(A/B)$ is equal to

(*a*) $\frac{14}{17}$ (*b*) $\frac{17}{20}$ (*c*) $\frac{7}{8}$ (*d*) $\frac{1}{8}$

**Sol.** (*a*), as $P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{70}{100} \times \frac{100}{85} = \frac{14}{17}$

**9.** $A$ speaks truth in 70% cases and $B$ speaks truth in 85% cases. The probability that they speak the same fact is

(*a*) 36% (*b*) 64% (*c*) 52% (*d*) 48%

**Sol.** (*b*), as $P(\text{same fact}) = P(AB \text{ or } \bar{A}\,\bar{B}) = \frac{70}{100} \times \frac{85}{100} + \frac{30}{100} \times \frac{15}{100} = \frac{5950 + 450}{10000} = \frac{6400}{10000} = 64\%$

**10.** Two dice are thrown once. If it is known that the sum of the numbers on the dice was less than 6 the probability of getting a sum 3 is

(*a*) $\frac{1}{18}$ (*b*) $\frac{5}{18}$ (*c*) $\frac{1}{5}$ (*d*) $\frac{2}{5}$

**Sol.** (*c*), as favourable cases for sum less than 6 are 10 and favourable for a total of 3 is 2.

**11.** A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is **[CBSE 2020]**

(*a*) $\frac{1}{3}$ (*b*) $\frac{4}{13}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{1}{2}$

**Sol.** (*c*), $P\left(\frac{\text{Queen of spade}}{\text{queen}}\right) = \frac{\frac{1}{52}}{\frac{4}{52}} = \frac{1}{4}$.

**12.** The value of $k$, for which the following distribution is a probability distribution

| $X$ | 30 | 10 | –10 |
|---|---|---|---|
| $P(X)$ | $\frac{1}{5}$ | $\frac{3}{10}$ | $k$ |

is

(*a*) $\frac{1}{3}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{1}{10}$ (*d*) $\frac{1}{5}$

**Sol.** (*b*), as for probability distribution

$$\Sigma P(X) = 1 \Rightarrow \frac{1}{5} + \frac{3}{10} + k = 1 \Rightarrow 1 - \frac{1}{2} = \frac{1}{2}.$$

**13.** Three balls are drawn from a bag containing 2 red and 5 black balls, if the random variable $X$ represents the number of red balls drawn, then $X$ can take values

(*a*) 0, 1, 2 (*b*) 0, 1, 2, 3 (*c*) 0 (*d*) 1, 2

**Sol.** (*a*), as there are 2 red balls, so maximum red balls can be 2.

**14.** Two events $A$ and $B$ are said to be independent if

(*a*) $P(A \cup B) = P(A) P(B)$ (*b*) $P(A \cap B) = 0$

(*c*) $P(A \cap B) = P(A) P(B)$ (*d*) none of these **[KVS]**

**Sol.** (*c*), result

**15.** If events $A$ and $B$ are independent, $P(A) = 0.35$, $P(A \cup B) = 0.60$ then $P(B)$ is

(*a*) 0.25 (*b*) 0 (*c*) 0.95 (*d*) none of these

**Sol.** (*d*), as $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$ (for independent and events)

$$\Rightarrow \quad 0.6 = 0.35 + P(B) - 0.35 \times P(B)$$

$$= 0.35 + 0.65\, P(B)$$

$$\Rightarrow \quad P(B) = \frac{0.25}{0.65} = 0.38$$

**16.** The probability of $A$, $B$ and $C$ solving a problem are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. Then the probability that the problem will be solved is

(*a*) $\frac{1}{2}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{4}$ (*d*) none of these

**Sol.** (*b*), as $P$(problem solved) = $1 - P$(none solves)

$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

**17.** A man is known to speak truth in 3 out of 4 times. He throws a dice and reports that it is a six. Then the probability that it is actually a six is

(*a*) $\frac{4}{9}$ (*b*) $\frac{2}{7}$ (*c*) $\frac{1}{6}$ (*d*) $\frac{3}{8}$

**Sol.** (*d*), as probability $= \dfrac{\frac{3}{4} \cdot \frac{1}{6}}{\frac{3}{4} \cdot \frac{1}{6} + \frac{1}{4} \cdot \frac{5}{6}} = \frac{3}{8}.$

**18.** A pair of dice is thrown and it is known that the second dice always exhibits an odd number. Then the probability that the sum obtained on two dice is 7, is

(*a*) $\frac{1}{6}$ (*b*) $\frac{5}{6}$ (*c*) $\frac{1}{2}$ (*d*) none of these

**Sol.** (*a*), as sum is 7 → {(1,6) (2,5) (3,4) (4,3) (5,4) (6,1)}

second dice odd no → {(1,1) (2,1) (3,1) (4,1) (5,1) (6,1)
(1,3) (2,3) (3,3) (4,3) (5,3) (6,3)
(1,5) (2,5) (3,5) (4,5) (5,5) (6,5)}

Probability $= \frac{3}{18} = \frac{1}{6}$

**19.** Bag *A* contains 3 red and 5 black balls and bag *B* contains 2 red and 4 black balls. A ball is drawn from one of the bags. The probability that ball drawn is red is

(*a*) $\frac{17}{24}$ (*b*) $\frac{17}{48}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{1}{3}$

**Sol.** (*b*) as $P(\text{red}) = P(A) \cdot P(R/A) + P(B) \cdot P(R/B) = \frac{1}{2} \cdot \frac{3}{8} + \frac{1}{2} \cdot \frac{2}{6} = \frac{3}{16} + \frac{1}{6} = \frac{17}{48}$.

**20.** A random variable has the following probability distribution:

| $X$ | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
|---|---|---|---|---|---|---|---|---|
| $P(X)$ | 0 | $2p$ | $2p$ | $3p$ | $p^2$ | $2p^2$ | $7p^2$ | $2p$ |

The value of $p$ is

(*a*) $-1$ (*b*) $\frac{1}{5}$ (*c*) $\frac{1}{10}$ (*d*) $\frac{3}{7}$

**Sol.** (*c*), as this represents a probability distributions

$$\therefore \quad \Sigma P(X) = 1$$
$$\Rightarrow \quad 0 + 2p + 2p + 3p + p^2 + 2p^2 + 7p^2 + 2p = 1$$
$$\Rightarrow \quad 10p^2 + 9p - 1 = 0$$
$$\Rightarrow \quad 10p^2 + 10p - p - 1 = 0$$
$$\Rightarrow \quad 10p(p + 1) - 1\,(p + 1) = 0$$
$$\Rightarrow \quad (10p - 1)\,(p + 1) = 0$$
$$\Rightarrow \quad 10p - 1 = 0 \text{ or } p + 1 = 0$$
$$\Rightarrow \quad p = \frac{1}{10} \text{ or } p = -1 \text{ (rejected)}$$
$$\therefore \quad p = \frac{1}{10}.$$

**21.** A bag contains 5 red, 6 blue and 4 black balls. Three balls are drawn from the bag. Then the probability that none of them is red, is

(*a*) $\frac{24}{91}$ (*b*) $\frac{2}{91}$ (*c*) $\frac{6}{35}$ (*d*) none of these

**Sol.** (*a*)

**22.** If $P(A) = 0.3$, $P(B) = 0.5$ and $P(A/B) = 0.4$, then $P(B/A)$ is

(*a*) $-\frac{2}{3}$ (*b*) $\frac{2}{3}$ (*c*) $\frac{3}{5}$ (*d*) none of these

**Sol.** (*b*)

**23.** A bag *A* contains 3 white, 2 red balls and a bag *B* contains 4 white and 5 red balls. One ball is drawn at random from one of the bags and is found to be red, then the probability that it was drawn from bag *B* is

(*a*) $\frac{27}{43}$ (*b*) $\frac{20}{43}$ (*c*) $\frac{25}{43}$ (*d*) none of these

**Sol.** (*c*)

**24.** A die is thrown three times, if the first throw results in 4, then the probability of getting 15 as a sum is

(*a*) $\frac{1}{3}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{1}{9}$ (*d*) $\frac{1}{18}$

**Sol.** (*d*)

**25.** Let *A* and *B* be the two events such that $P(A) = 0.2$, $P(B) = 0.3$ and $P(A/B) = 0.5$ then $P(A'/B')$ is

(*a*) 0.15 (*b*) 0.35 (*c*) $\frac{13}{14}$ (*d*) 0.70

**Sol.** (*c*), as $P(A/B) = \dfrac{P(A \cap B)}{P(B)} \Rightarrow P(A \cap B) = 0.5 \times 0.3 = 0.15$

$$\text{Now } P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A) - P(B) + P(A \cap B)}{1 - P(B)}$$

$$= \frac{1 - 0.2 - 0.3 + 0.15}{1 - 0.3} = \frac{0.65}{0.7} = \frac{65}{70} = \frac{13}{14}.$$

**26.** If the probability of happening of at least one of the two events *A* or *B* is *p* and probability of their simultaneous happening is *q*, then the value of $P(\overline{A}) + P(\overline{B})$ is

(*a*) $p + q - 2$ (*b*) $2 - p - q$ (*c*) $2 - p + q$ (*d*) $2 + p - q$

**Sol.** (*b*), as

$$\begin{aligned} P(\overline{A}) + P(\overline{B}) &= 1 - P(A) + 1 - P(B) \\ &= 2 - [P(A) + P(B)] \\ &= 2 - [P(A \cup B) + P(A \cap B)] \\ &= 2 - (p + q) = 2 - p - q \end{aligned}$$

**27.** Let *E* and *F* be the events with $P(E) = \frac{3}{5}$, $P(F) = \frac{1}{3}$, and $P(E \cap F) = \frac{1}{5}$, then events *E* and *F* are

(*a*) mutually exclusive (*b*) independent events

(*c*) exhaustive events (*d*) dependent events

**Sol.** (*b*), As $P(E) \cdot P(F) = \frac{3}{5} \times \frac{1}{3} = \frac{1}{5} = P(E \cap F)$. Hence, independent.

**28.** Given $P(A) = 0.3$, $P(B) = 0.8$ and $P(B/A) = 0.5$, then $P(A \cap \overline{B})$ is

(*a*) 0.45 (*b*) 0.35 (*c*) 0.65 (*d*) 0.15

**Sol.** (*d*), As $P(B/A) = \dfrac{P(A \cap B)}{P(A)} \Rightarrow P(A \cap B) = 0.5 \times 0.3 = 0.15$

$$P(A \cap \overline{B}) = P(A) - P(A \cap B) = 0.3 - 0.15 = 0.15$$

**29.** $A$ and $B$ are two given events such that $P(A) \neq 0$. If $A$ is a subset of $B$ then $P(B/A)$ is

(*a*) 1 (*b*) 0 (*c*) $P(A)$ (*d*) $P(B)$

**Sol.** (*a*), as $A \subseteq B \Rightarrow P(A \cap B) = P(A)$

$$P(B/A) = \frac{P(\text{A} \cap \text{B})}{P(A)} = \frac{P(A)}{P(A)} = 1$$

**30.** Mother, father and son line up at random for a family picture. If E is the event 'Son on one end' and F is the event 'Father in the middle', then $P(E/F)$ is

(*a*) $\frac{1}{2}$ (*b*) 1 (*c*) 2 (*d*) $\frac{1}{3}$

**Sol.** (*b*) As $E$ : son on one end *MFS, FMS, SFM, SMF*

$F$ : Father in the middle *MFS, SFM*

$E \cap F$ : *MFS, SFM*

$$\therefore \quad P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{n(E \cap F)}{n(F)} = \frac{2}{2} = 1$$

**31.** Given $P(\overline{A}) = 0.4$, $P(B) = 0.2$ and $P(A/B) = 0.5$, then $P(A \cup \overline{B})$ is

(*a*) 0.35 (*b*) 0.9 (*c*) 0.65 (*d*) 0.55

**Sol.** (*b*), as $P(A/B) = \dfrac{P(A \cap B)}{P(B)}$

$$\Rightarrow \quad P(A \cap B) = 0.5 \times 0.2 = 0.10$$

$$\begin{aligned} P(A \cup \overline{B}) &= P(A) + P(\overline{B}) - P(A \cap \overline{B}) \\ &= P(A) + 1 - P(B) - P(A) + P(A \cap B) \\ &= 1 - P(B) + P(A \cap B) \\ &= 1 - 0.2 + 0.1 \\ &= 0.9 \end{aligned}$$

## Assertion-Reason Questions

(*a*) If both 'A' and 'R' are true and R is the correct explanation of A

(*b*) If both 'A' and 'R' are true and R is not the correct explanation of A

(*c*) A is true R, is false

(*d*) A is false, R is true

**32. Assertion (A):** Two dice are rolled and it is given that the sum of the number on both dice is greater than 6. The Probability of getting a doublet is $\frac{3}{16}$.

**Reason (R):** Probability of an event $E$ when $F$ is given is $P(E/F) = \frac{P(E \cap F)}{P(F)}$

**Sol.** (*d*)

Total outcomes = 36

Let $E$ =coming of doublet, {(1, 1), (2, 2), (3, 3), (4, 4), (5, 5), (6, 6)}

$F$ = coming the numbers whose sum is greater than 6

{(1, 6), (2, 5), (2, 6), (3, 4), (3, 5), (3, 6), (4, 3), (4, 4), (4, 5), (4, 6), (5, 1), (5, 2), (5, 3), (5, 4), (5, 5), (5, 6), (6, 1), (6, 2), (6, 3), (6, 4), (6, 5), (6, 6)}

$$E \cap F = \{(4, 4), (5, 5), (6, 6)\}$$

$$P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{3/36}{22/36} = \frac{3}{22}$$

A is false.

R is true.

**33. Assertion (A):** If $A$ and $B$ are independent event then $P(A \cap B)' = P(A)' \cdot P(B)'$

**Reason (R):** $P(A \cup B)' = 1 - P(A \cup B)$

**Sol.** (*d*)

$$P(A \cap B)' = P(A' \cup B')$$
$$= P(A') + P(B') - P(A' \cap B')$$
$$= P(A') + P(B') - P(A') \cdot P(B')$$

A, is not true.

Now $P(A \cup B)' = 1 - P(A \cup B)$ is true.

R, is true.

**34. Assertion (A):** All complementary events are mutually exclusive events.

**Reason (R):** The sum of probability of two mutually exclusive events is 1.

**Sol.** (*c*)

For complementary events $A \cap A' = \phi$

and for two mutually exclusive events

$$A \cap B = \phi$$

So, A is true.

Now if

$$S = \{1, 2, 3, 4, 5, 6\}$$
$$E = \{1, 2\}$$
$$F = \{4, 5\}$$

Here $E \cap F = \phi$. So they are mutually exclusive but $P(E) + P(F) = \frac{1}{3}+\frac{1}{3}=\frac{2}{3} \neq 1$.

So, R is false.

**35. Assertion (A):** If one card is lost from the pack of 52 cards and a card is drawn from the remaining cards. The probability that the drawn card is diamond is $\frac{1}{4}$.

**Reason (R):** For two independent events $A$ and $B$, $P(A \cup B) = P(A) + P(B) - P(A) \cdot P(B)$

**Sol.** (*b*)

$$E_1 = \text{Lost card is diamond,}$$
$$P(E_1) = \frac{1}{4}$$
$$E_2 = \text{lost card is not diamond}$$
$$P(E_2) = \frac{3}{4}$$
$$A = \text{drawn card is diamond}$$
$$P(\text{A/E}_1) = \frac{12}{51},\ P(A/E_2) = \frac{13}{51}$$
$$P(A) = P(A/E_1)\cdot P(E_1) + P(A/E_2)\cdot P(E_2) = \frac{12}{51}\times\frac{1}{4}+\frac{13}{51}\times\frac{3}{4} = \frac{51}{51\times 4} = \frac{1}{4}$$

A is true.

For two independent events $A$ and $B$

$$P(A \cap B) = P(A)\cdot P(B)$$
$$\Rightarrow \quad P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= P(A) + P(B) - P(A)\cdot P(B)$$

R is also true but not the correct explanation of A.

**36. Assertion (A):** If Ram does 60% work of a company with 90% accuracy and the remaining work is done by Shyam with 95% accuracy. If the work done is found to be not accurate, then the probability that the work is done by Ram is $\frac{3}{4}$.

**Reason (R):** If $P(A/E_1)$ and $P(A/E_2)$ are the conditional probability of an event under conditions $E_1$ and $E_2$ then

$$P(E_1/A) = \frac{P(A/E_1)\cdot P(E_1)}{P(A/E_1)\cdot P(E_1)+P(A/E_2)\cdot P(E_2)}$$

**Sol.** (*a*)

$$E_1 = \text{Work is done by Ram}$$
$$E_2 = \text{Work is done by Shyam}$$
$$P(E_1) = 0.6,\ P(E_2) = 0.4$$

$$A = \text{Work is not accurate}$$

$$P(A/E_1) = 0.10,\ P(A/E_2) = 0.05$$

$$P(E_1/A) = \frac{P(A/E_1) \times P(E_1)}{P(A/E_1) \cdot P(E_1) + P(A/E_2) \cdot P(E_2)} = \frac{0.10 \times 0.6}{0.10 \times 0.6 + 0.05 \times 0.4} = \frac{3}{4}$$

R is also true and correct explanation of A.

## Case-based Questions

**37.** A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots

From this situation answer the following:

(*i*) Let the target is hit by $A$, $B$: the target is hit by $B$ and, $C$: the target is hit by $A$ and $C$. Then, the probability that $A$, $B$ and, $C$ all will hit, is

(*a*) $\frac{4}{5}$ (*b*) $\frac{3}{5}$ (*c*) $\frac{2}{5}$ (*d*) $\frac{1}{5}$

(*ii*) Referring to (*i*), what is the probability that $B$, $C$ will hit and $A$ will lose?

(*a*) $\frac{1}{10}$ (*b*) $\frac{3}{10}$ (*c*) $\frac{7}{10}$ (*d*) $\frac{4}{10}$

(*iii*) With reference to the events mentioned in (*i*), what is the probability that 'any two of $A$, $B$ and $C$ will hit?

(*a*) $\frac{1}{30}$ (*b*) $\frac{11}{30}$ (*c*) $\frac{17}{30}$ (*d*) $\frac{13}{30}$

(*iv*) What is the probability that 'none of them will hit the target'?

(*a*) $\frac{1}{30}$ (*b*) $\frac{1}{60}$ (*c*) $\frac{1}{15}$ (*d*) $\frac{2}{15}$

(*v*) What is the probability that at least one of $A$, $B$ or $C$ will hit the target?

(*a*) $\frac{59}{60}$ (*b*) $\frac{2}{5}$ (*c*) $\frac{3}{5}$ (*d*) $\frac{1}{60}$

**Sol.** (*i*) (*c*), $\frac{2}{5}$ (*ii*) (*a*), $\frac{1}{10}$ (*iii*) (*d*), $\frac{13}{30}$ (*iv*) (*b*), $\frac{1}{60}$ (*v*) (*a*), $\frac{59}{60}$

**38.** Ajay and Rakhi decided to play with dice at home during summer holidays. Ajay throws a dice repeatedly until a six is obtain. He denote the number of throws required by A. Based on this information, answer the following questions.

(*i*) The probability that $A = 5$ equals

(*a*) $\frac{1}{6^6}$ (*b*) $\frac{1}{6^4}$ (*c*) $\frac{5}{6^4}$ (*d*) $\frac{5^4}{6^5}$

(*ii*) The probability that $A = 3$ equals

(*a*) $\frac{5}{3^6}$ (*b*) $\frac{1}{6^3}$ (*c*) $\frac{1}{6}$ (*d*) $\frac{5^2}{6^3}$

(*iii*) The probability that $A \geq 3$ equals

(*a*) $\frac{5}{36}$ (*b*) $\frac{25}{216}$ (*c*) $\frac{25}{36}$ (*d*) $\frac{25}{136}$

(*iv*) The conditional probability that $A \geq 6$ given $A > 3$ equals

(*a*) $\frac{5}{6}$ (*b*) $\frac{26}{25}$ (*c*) $\frac{1}{6}$ (*d*) $\frac{5^2}{6^2}$

(*v*) The value of $P(A > 3) + P(A \geq 6)$ is

(*a*) $\frac{5^3 \times 61}{6^5}$ (*b*) $\frac{5^3}{6^4}$ (*c*) $\frac{1-5^3}{6^5}$ (*d*) $\frac{5^3}{6^5}$

**Sol.** (*i*) (*d*), $P(A = 5) = \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{5^4}{6^5}$

(*ii*) (*d*), $P(A = 3)$ = (Probability of not getting six at first chance) × (Probability of not gets six at second chance) × (Probability of getting six at third chance)

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \frac{5^2}{6^3}$$

(*iii*) (*c*), $P(A \geq 3) = 1 - P(A < 3) = 1 - [P(A = 1) + P(A = 2)]$

$$= 1 - \left[\frac{1}{6} + \frac{5}{6} \times \frac{1}{6}\right] = 1 - \frac{11}{36} = \frac{25}{36}$$

(*iv*) (*d*), $P(A \geq 6) = \left(\frac{5}{6}\right)^5 \times \frac{1}{6} + \left(\frac{5}{6}\right)^6 \times \frac{1}{6} + \ldots\infty$

$$= \frac{5^5}{6^6}\left[1 + \frac{5}{6} + \left(\frac{5}{6}\right)^2 + \ldots + \infty\right] = \frac{5^5}{6^6}\left[\frac{1}{1 - \frac{5}{6}}\right] = \left(\frac{5}{6}\right)^5$$

$$P(A > 3) = \left(\frac{5}{6}\right)^3 \times \frac{1}{6} + \left(\frac{5}{6}\right)^4 \times \frac{1}{6} + \ldots\infty$$

$$= \left(\frac{5}{6}\right)^3 \times \frac{1}{6}\left[1 + \frac{5}{6} + \ldots\right]$$

$$= \frac{5^3}{6^4} \times \left[\frac{1}{1 - \frac{5}{6}}\right] = \left(\frac{5}{6}\right)^3$$

$$\text{Required conditional probability} = \frac{\left(\frac{5}{6}\right)^5}{\left(\frac{5}{6}\right)^3} = \frac{5^2}{6^2}$$

(*v*) (*a*), $P(A > 3) + (P \geq 6) = \left(\frac{5}{6}\right)^3 + \left(\frac{5}{6}\right)^5$

$$= \left(\frac{5}{6}\right)^3\left[\frac{5^2}{6^2} + 1\right] = \frac{5^3 \times 61}{6^5}$$

**39.** Nisha and Arun appeared for first round of an competitive examination for two vacancies. The probability of Nisha's selection is $\frac{1}{6}$ and that of Arun's selection is $\frac{1}{4}$.

Based on the above information, answer the following questions:

(*i*) The probability that at least one of them is selected, is

(*a*) $\frac{1}{8}$ (*b*) $\frac{3}{7}$ (*c*) $\frac{2}{7}$ (*d*) $\frac{3}{8}$

(*ii*) The probability that both of them are selected is

(*a*) $\frac{1}{6}$ (*b*) $\frac{1}{12}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{24}$

(*iii*) The probability that none of them is selected, is

(*a*) $\frac{3}{8}$ (*b*) $\frac{1}{3}$ (*c*) $\frac{2}{7}$ (*d*) $\frac{5}{8}$

(*iv*) The probability that only one of them is selected, is

(*a*) $\frac{2}{7}$ (*b*) $\frac{5}{8}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{1}{3}$

(*v*) Suppose Nisha is selected by the director and told her about two posts *X* and *Y* for which posting is independent. If the probability of posting for post X is $\frac{1}{6}$ and for post Y is $\frac{1}{7}$, then the probability that Nisha is selected for at least one post, is

(*a*) $\frac{3}{8}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{2}{7}$ (*d*) $\frac{3}{7}$

**Sol.** Let *M* be the event that Nisha is selected and *Q* be the event that Arun is selected. Then we have

$$P(M) = \frac{1}{6}$$

$$\Rightarrow \quad P(\overline{M}) = 1 - \frac{1}{6} = \frac{5}{6} = P(\text{Nisha is not selected})$$

$$\Rightarrow \quad P(Q) = \frac{1}{4}$$

$$\Rightarrow \quad P(\overline{Q}) = 1 - \frac{1}{4} = \frac{3}{4} = P(\text{Arun is not selected})$$

(*i*) (*d*), *P*(at least one of them is selected)

$$= 1 - P(\text{both are rejected}) = 1 - P(\overline{M}) \cdot P(\overline{Q})$$

$$= 1 - \frac{5}{6} \times \frac{3}{4} = 1 - \frac{5}{8} = \frac{3}{8}$$

(*ii*) (*d*), Selection is independent

$$P(\text{both are selected}) = P(M \cap Q)$$

$$= P(M) \cdot P(Q) = \frac{1}{6} \times \frac{1}{4} = \frac{1}{24}$$

(*iii*) (*d*), $P(\text{none of them is selected}) = P(\text{both are rejected})$

$$= P(\overline{M}) \cdot P(\overline{Q}) = \frac{5}{6} \times \frac{3}{4} = \frac{5}{8}$$

(*iv*) (*d*), $P(\text{only one of them is selected}) = P(M \cap \overline{Q}) + P(\overline{M} \cap Q)$

$$= P(M) \cdot P(\overline{Q}) + P(\overline{M}) \cdot P(Q)$$

$$= \frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4} = \frac{3}{24} + \frac{5}{24} = \frac{8}{24} = \frac{1}{3}$$

(*v*) (*c*), Let $E_1$ be the event that Nisha is selected for Post *X* and $E_2$ be the event that Nisha is selected for post *Y*.

$\therefore$ $P(\text{Nisha is selected for at least one post})$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{1}{6} + \frac{1}{7} - \frac{1}{6} \times \frac{1}{7} = \frac{12}{42} = \frac{2}{7}$$

**40.** In an office three employees *A*, *B* and *C* are deputed to check the quality of product produced. *A*, *B* and *C* process 50%, 20%, 30% of the product respectively. It was found that 6% product by *A*, 4% by *B* and 3% by *C* is found to be below quality (*E*). Based on the above information answer the following:

(*i*) The probability of *B* checking the quality of the product is

(*a*) $\frac{3}{5}$ (*b*) $\frac{1}{5}$ (*c*) $\frac{2}{5}$ (*d*) $\frac{7}{5}$

(*ii*) The probability of product below quality processed by *A* is

(*a*) 0.5 (*b*) 0.06 (*c*) 0.2 (*d*) 0.03

(*iii*) The probability that product is below quality is

(*a*) 0.0389 (*b*) 0.047 (*c*) 0.35 (*d*) 0.15

(*iv*) The probability of quality of product checked by *A*, *B* or *C* is

(*a*) 1 (*b*) 0.5 (*c*) 0.2 (*d*) 0.3

(*v*) The probability that the selected product below quality is checked by *C* is

(*a*) $\frac{9}{470}$ (*b*) $\frac{3}{470}$ (*c*) $\frac{3}{47}$ (*d*) $\frac{9}{47}$

**Sol.** (*i*) (*b*), Probability of *B* checking the quality $= \frac{20}{100} = \frac{1}{5}$

(*ii*) (*d*), $P(A) \cdot P(E/A) = \frac{50}{100} \times \frac{6}{100} = 0.03$

$E$ : below quality

(*iii*) (*b*), Product below quality

$$P(E) = P(A) \cdot P(E/A) + P(B) \cdot P(E/B) + P(C) \cdot P(E/C)$$
$$= \frac{50}{100} \times \frac{6}{100} + \frac{20}{100} \times \frac{4}{100} + \frac{30}{100} \times \frac{3}{100}$$
$$= \frac{470}{10000} = 0.047$$

(*iv*) (*a*), $P(A) + P(B) + P(C) = 1$

(*v*) (*d*), Using Baye's Theorem

$$P(C/E) = \frac{P(C) \cdot P(E/C)}{P(E)} = \frac{\frac{30}{100} \times \frac{3}{100}}{0.047} = \frac{9}{47}$$

**41.** An building contractor undertakes a job to construct 4 flats on a plot along with parking area. The probability of all construction workers not present for the job is 0.65. If they are not present and still work gets completed is 0.35. The probability that work will be completed when all workers are present is 0.80. Based on the above information answer the following: if $E_1$ represent the event when all workers were not present for the job and $E_2$ represents event when workers were present, $E$ represents completing the construction work on time.

(*i*) What is the probability that workers are present for the job?

(*a*) 0.65 (*b*) 1 (*c*) 0.35 (*d*) 0.80

(*ii*) What is the probability that construction will be completed on time?

(*a*) 0.51 (*b*) 0.60 (*c*) 0.39 (*d*) 0.45

(*iii*) What is the probability that workers are not present given that construction work is completed on time?

(*a*) 0.54 (*b*) 0.40 (*c*) 0.49 (*d*) 0.45

(*iv*) What is the probability that workers were present given that construction job was completed on time?

(*a*) 0.55 (*b*) 0.50 (*c*) 0.45 (*d*) 0.73

(*v*) Find the value of $\sum_{i=1}^{2} P(E_i/E)$ :

(*a*) 0.51 (*b*) 0.60 (*c*) 1 (*d*) 0.93

**Sol.** (*i*) (*c*), $P(E_2) = 1 - P(E_1) = 1 - 0.65 = 0.35$

(*ii*) (*a*), $P(E) = P(E_1) \cdot P(E/E_1) + P(E_2) \cdot P(E/E_2)$

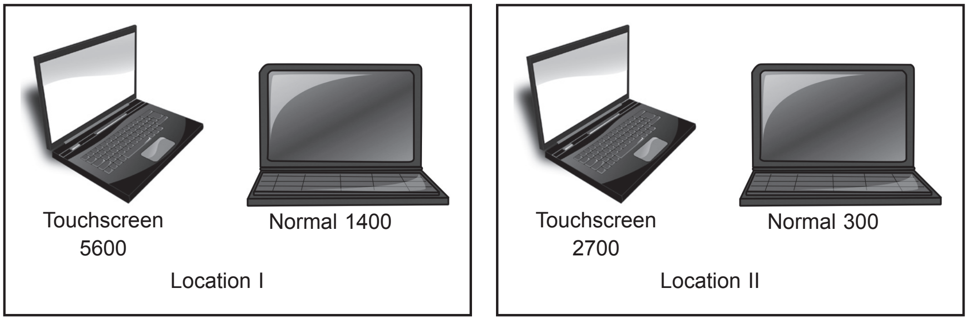
$$= 0.65 \times 0.35 + 0.35 \times 0.80 = 0.35 \times 1.45 = 0.51$$

(*iii*) (*d*), $P(E_1/E) = \frac{P(E_1) \cdot P(E/E_1)}{P(E)} = \frac{0.65 \times 0.35}{0.51} = 0.45$

(*iv*) (*a*), $P(E_2/E) = \frac{P(E_2) \cdot P(E/E_2)}{P(E)} = \frac{0.35 \times 0.80}{0.51} = 0.55$

(*v*) (*c*).

**42.** A manufacturing company manufactures laptops at two plants located at different locations. The laptops manufactured are of two types one is touchscreen, other one is normal laptop. Production in two plants is shown as




A laptop is chosen at random and is found to be touchscreen Laptop. Based on above information. Answer the following:

(*i*) What is the probability that selected laptop is produced at location I?

(*a*) $\frac{7}{8}$ (*b*) $\frac{8}{70}$ (*c*) $\frac{4}{5}$ (*d*) $\frac{3}{10}$

(*ii*) What is the probability that selected laptop is produced at location II?

(*a*) $\frac{7}{8}$ (*b*) $\frac{8}{70}$ (*c*) $\frac{7}{10}$ (*d*) $\frac{9}{10}$

(*iii*) Find the probability of choosing a touchscreen laptop.

(*a*) $\frac{17}{20}$ (*b*) $\frac{85}{50}$ (*c*) $\frac{2}{5}$ (*d*) $\frac{9}{20}$

(*iv*) What is the probability of choosing a normal laptop?

(*a*) $\frac{7}{20}$ (*b*) $\frac{3}{20}$ (*c*) $\frac{1}{10}$ (*d*) $\frac{1}{20}$

(*v*) What is the probability that chosen touchscreen laptop is produced at plant II?

(*a*) $\frac{5}{27}$ (*b*) $\frac{17}{20}$ (*c*) $\frac{9}{17}$ (*d*) $\frac{9}{20}$

**Sol.** (*i*) (*c*)

(*ii*) (*d*)

(*iii*) (*a*), $P(\text{touchscreen laptop}) = \frac{1}{2}\cdot\frac{4}{5}+\frac{1}{2}\cdot\frac{9}{10}=\frac{17}{20}$

(*iv*) (*b*), $P(\text{normal laptop}) = \frac{1}{2}\cdot\frac{1}{5}+\frac{1}{2}\cdot\frac{1}{10}=\frac{3}{20}$

(*v*) (*c*), Probability of chosen laptop is produced in plant II

$$=\frac{\frac{1}{2}\times\frac{9}{10}}{\frac{1}{2}\times\frac{4}{5}+\frac{1}{2}\times\frac{9}{10}}=\frac{9}{17}$$

**43.** Two friends $A$ and $B$ had gone for a shopping, during shopping they came across a beautiful antique piece and both wanted to purchase it. They asked the shopkeeper for another piece so that both can purchase one-one. But shopkeeper replied he had only one piece. Both the friends didn't want to attend each other, so they decided a way out. They said lets go to a coffee shop have coffee and at the sametime toss a pair of coins. Whosoever gets the pair of heads first will buy the antique piece, both shook hands and sat down for their luck as they have coffee you answer the following questions, if $A$ starts

(*i*) What is the probability of getting a pair of heads?

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{8}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{3}{4}$

(*ii*) What is the probability of getting only one head in a throw?

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{8}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{3}{4}$

(*iii*) What is the probability that $A$ gets a pair of heads in third throw and wins the game?

(*a*) $\frac{9}{16}$ (*b*) $\frac{3}{64}$ (*c*) $\frac{9}{64}$ (*d*) $\frac{1}{16}$

(*iv*) What is the probability that $B$ wins the game, if $A$ starts?

(*a*) $\frac{3}{8}$ (*b*) $\frac{3}{7}$ (*c*) $\frac{2}{9}$ (*d*) $\frac{5}{7}$

(*v*) What is the probability that $A$ buys the antique piece, if $A$ starts?

(*a*) $\frac{3}{8}$ (*b*) $\frac{3}{7}$ (*c*) $\frac{2}{9}$ (*d*) $\frac{4}{7}$

**Sol.** (*i*) (*c*), $\{HH\}$, $\frac{1}{4}$

(*ii*) (*a*), $\{HT, TH\}$, $\frac{2}{4} = \frac{1}{2}$

(*iii*) (*c*), $P(\text{pair of heads in third throw}) = P(\overline{A})\, P(\overline{B})\, P(A) = \frac{3}{4} \times \frac{3}{4} \times \frac{1}{4} = \frac{9}{64}$

(*iv*) (*b*),

$$P(B) = P(\overline{A})\, P(B) + P(\overline{A})\, P(\overline{B})\, P(\overline{A})\, P(B) + \ldots$$

$$= \frac{3}{4} \cdot \frac{1}{4} + \left(\frac{3}{4}\right)^3 \cdot \frac{1}{4} + \ldots = \frac{\frac{3}{16}}{1 - \frac{9}{16}} = \frac{3}{7}$$

(*v*) (*d*),

$$P(A) = P(A) + P(\overline{A})\, P(\overline{B})\, P(A) + \ldots$$

$$= \frac{1}{4} + \left(\frac{3}{4}\right)^2 \cdot \frac{1}{4} + \ldots = \frac{\frac{1}{4}}{1 - \frac{9}{16}} = \frac{4}{7}.$$

**44.** A student has to appear in an examination in two subjects $A$ and $B$. He had prepared for the subject but not so thoroughly, so he was guessing about passing or failing in the subject. So he thought the probability for both the subjects as $P(A \text{ fails}) = 0.12$, $P(\text{fails in } B \text{ alone}) = 0.15$, $P(\text{failing in } A \text{ and } B \text{ both}) = 0.06$

Based on above answer the following:

(*i*) What the probability of failing in subject *B*?

(*a*) 0.15 (*b*) 0.12 (*c*) 0.21 (*d*) 0.42

(*ii*) What is the probability of failing in subject *A*, given that he has already failed in subject *B*?

(*a*) $\frac{2}{5}$ (*b*) $\frac{2}{7}$ (*c*) $\frac{3}{5}$ (*d*) $\frac{3}{7}$

(*iii*) What is the probability of failing in subject *B*, given that he has failed in subject *A*?

(*a*) $\frac{1}{2}$ (*b*) $\frac{2}{7}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{3}{5}$

(*iv*) What is the probability of failing in subject *A* or subject *B*?

(*a*) 0.21 (*b*) 0.33 (*c*) 0.15 (*d*) 0.27

(*v*) What is the probability of neither fail in *A* nor in *B*?

(*a*) 0.83 (*b*) 0.73 (*c*) 0.79 (*d*) 0.77

**Sol.** (*i*) (*c*), $P(B) = P(\text{only } B) + P(A \cap B) = 0.15 + 0.06 = 0.21$

(*ii*) (*b*), $P(A/B) = \dfrac{P(A \cap B)}{P(B)} = \dfrac{0.06}{0.21} = \dfrac{6}{21} = \dfrac{2}{7}$

(*iii*) (*a*), $P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.06}{0.12} = \dfrac{6}{12} = \dfrac{1}{2}$

(*iv*) (*d*), $P(A \text{ or } B) = P(A) + P(B) - P(A \cap B)$

$= 0.12 + 0.21 - 0.06 = 0.27$

(*v*) (*b*), $P(\text{neither } A \text{ nor } B) = 1 - P(A \text{ or } B) = 1 - 0.27 = 0.73$

**45.** Year 2021 will be remembered as most challenging year due to COVID-19. Practically every person took precautions, some are shown as

Washing hands

Wearing mask

Sanitize place

Keep distance

and our medical staff worked 24 hours to contain it. Lot of tests were performed and under are such survey of the people having covid-19, 90% of the tests detected the disease and 10% go unnoticed. Of the people free of covid-19. 99% are judged covid-19 negative but 1% are shown as covid positive. From a large population only 1 person in 1000 is covid-19 positive. One person is selected at random and test is performed for covid-19 and is found to be positive.

Based on above information, answer the following:

(*i*) What is the probability of the person tested is covid-19 positive, given that he actually suffers from covid-19?

(*a*) 0.9 (*b*) 0.001 (*c*) 0.1 (*d*) 0.99

(*ii*) What is the probability of the person tested as covid-19 positive, given that he is not suffering from covid-19?

(*a*) 0.9 (*b*) 0.001 (*c*) 0.01 (*d*) 0.1

(*iii*) What is the probability that person is not suffering from covid-19?

(*a*) 0.001 (*b*) 0.9 (*c*) 0.01 (*d*) 0.999

(*iv*) What is the probability that person suffers from covid-19 given that he is tested covid-19 positive?

(*a*) $\frac{5}{121}$ (*b*) $\frac{10}{121}$ (*c*) $\frac{10}{111}$ (*d*) $\frac{4}{99}$

(*v*) What is the probability that person selected will be diagonised as covid positive?

(*a*) 0.019 (*b*) 0.2 (*c*) 0.1089 (*d*) 0.99

**Sol.** Let A: persons selected suffer from Covid-19, B: person selected does not suffer from Covid-19. E: person's Covid-19 test is positive.

(*i*) (*a*), $P(E/A) = 90\% = 0.9$

(*ii*) (*c*), $P(E/B) = 1\% = 0.01$

(*iii*) (*d*), $P(B) = 1 - \frac{1}{1000} = \frac{999}{1000} = 0.999$

(*iv*) (*b*), $$P(A/E) = \frac{P(A) \cdot P(E/A)}{P(A) \cdot P(E/A) + P(B) \cdot P(E/B)}$$

$$= \frac{(0.001) \cdot (0.9)}{(0.001) \cdot (0.9) + (0.999) \cdot (0.01)} = \frac{10}{121}$$

(*v*) (*c*), $$P(E) = P(A) \cdot P(E/A) + P(B) \cdot B(E/B)$$

$$= (0.9)\,(0.001) + (0.999)\,(0.01) = 0.1089$$

**46.** Three persons $A$, $B$ and $C$ apply for the job of a manager in a private company the chances of their selection is given by the relation $4A = 2B = C$. The probability that if selected $A$, $B$ and $C$ can bring changes to improve profitability of the company are 0.8, 0.5 and 0.3 respectively. Based on the above information answer the following:

(*i*) The probability that $C$ is selected as a manager is

(*a*) $\frac{1}{7}$ (*b*) $\frac{2}{7}$ (*c*) $\frac{3}{7}$ (*d*) $\frac{4}{7}$

(*ii*) What is conditional probability that if change has taken place it is due to $B$?

(*a*) 0.8 (*b*) 0.5 (*c*) 0.3 (*d*) 0.7

(*iii*) What is the conditional probability that change does not take place due to selection of $A$?

(*a*) 0.7 (*b*) 0.8 (*c*) 0.5 (*d*) 0.2

(*iv*) What is the probability that change does not take place?

(*a*) $\frac{2}{7}$ (*b*) $\frac{4}{7}$ (*c*) $\frac{3}{7}$ (*d*) $\frac{1}{7}$

(*v*) If change has not taken place what is probability that it was due to selection of $C$?

(*a*) $\frac{7}{10}$ (*b*) $\frac{1}{20}$ (*c*) $\frac{1}{80}$ (*d*) $\frac{4}{15}$

**Sol.** (*i*) (*d*),

$$4A = 2B = C \Rightarrow \frac{A}{1} = \frac{B}{2} = \frac{C}{4}$$

$$A : B : C = 1 : 2 : 4$$

$$P(C) = \frac{4}{7}$$

(*ii*) (*b*), 0.5

(*iii*) (*d*), $1 - 0.8 = 0.2$

(*iv*) (*b*), P(change does not take place)

If $E$ represents change then $P(E) = P(A) \cdot P(E/A) + P(B) \cdot P(E/B) + P(C) \cdot P(E/C)$

$$= \frac{1}{7} \times 0.8 + \frac{2}{7} \times 0.5 + \frac{4}{7} \times 0.3$$

$$= \frac{0.8 + 1.0 + 1.2}{7} = \frac{3}{7}$$

$$P(\overline{E}) = 1 - P(E) = 1 - \frac{3}{7} = \frac{4}{7}$$

(*v*) (*a*), Using Baye's theorem

$$P(C/\overline{E}) = \frac{\frac{4}{7} \times 0.7}{\frac{4}{7}} = \frac{7}{10}$$

**47.** During examinations, we need to reschedule on study hours and along with the study hours we need quality revision of syllabus. In one such situation if $X$ is random variable which represents number of hours a student of class XII studies a particular subject per day. The probability distribution is given as $P(X) = \begin{cases} 0, & \text{if } X = 0 \\ k, & \text{if } X = 1 \\ 3k, & \text{if } X = 2 \\ 4k, & \text{if } X = 3 \\ 0, & \text{if } X > 3 \end{cases}$

Based on the above information, answer the following:

(*i*) What is the value of $k$?

(*a*) 1 (*b*) $\frac{1}{6}$ (*c*) $\frac{1}{7}$ (*d*) $\frac{1}{8}$

(*ii*) The probability that less than 2 hours time is given to a subjects, per day is

(*a*) $\frac{1}{8}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{5}{8}$

(*iii*) What is the probability that 2 hours or 3 hours are given to a subject per day?

(*a*) $\frac{1}{8}$ (*b*) $\frac{7}{8}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{5}{8}$

(*iv*) What is the probability that 3 hours or more are given to a subject per day?

(*a*) $\frac{2}{5}$ (*b*) $\frac{1}{8}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{3}$

(*v*) What is the probability that one hour is given to a particular subject per day?

(*a*) $\frac{1}{4}$ (*b*) $\frac{1}{8}$ (*c*) $\frac{1}{2}$ (*d*) $\frac{3}{8}$

**Sol.** (*i*) (*d*), $\Sigma P(X) = 1 \Rightarrow P(0) + P(1) + P(2) + P(3) + P(4) + \ldots.. = 1$

$$\Rightarrow 0 + k + 3k + 4k + 0 + \ldots = 1 \Rightarrow 8k = 1 \Rightarrow k = \frac{1}{8}$$

(*ii*) (*a*), $P(X < 2) = P(0) + P(1) = 0 + k = k = \frac{1}{8}$

(*iii*) (*b*), $P(2) + P(3) = 3k + 4k = 7k = 7 \times \frac{1}{8} = \frac{7}{8}$

(*iv*) (*c*), $P(X \geq 3) = P(3) + P(4) + P(5) + \ldots.. = 4k + 0 + 0 + \ldots..$

$$= 4k = 4 \times \frac{1}{8} = \frac{1}{2}$$

(*v*) (*b*), $P(1) = k = \frac{1}{8}$

**48.** A company producing electric bulbs has factories at three locations $E_1$, $E_2$ and $E_3$ and company got a bulk order of producing electric bulbs. The capacities at locations $E_1$ and $E_3$ are same and at location $E_2$ is double that of $E_1$. Also it is known that 4% of bulbs produced at $E_1$ and $E_2$ are defective and 5% produced at $E_3$ are defective.

From the above information, answer the following:

(*i*) What is the probability of production capacity of $E_3$?

(*a*) $\frac{1}{2}$ (*b*) $\frac{1}{4}$ (*c*) $\frac{1}{3}$ (*d*) $\frac{1}{5}$

(*ii*) What is conditional probability of producing defective bulb by $E_2$?

(*a*) 96% (*b*) 4% (*c*) 5% (*d*) 95%

(*iii*) What is the probability of a defective bulb?

(*a*) $\frac{13}{400}$ (*b*) 0.01 (*c*) 0.02 (*d*) $\frac{17}{400}$

(*iv*) If a defective bulb has been produced, what is the probability that it is produced at location $E_2$?

(*a*) $\frac{1}{17}$ (*b*) $\frac{2}{13}$ (*c*) $\frac{4}{13}$ (*d*) $\frac{8}{17}$

(*v*) Which location produces least defective bulbs?

(*a*) $E_1$ (*b*) $E_2$ (*c*) $E_3$ (*d*) none of these

**Sol.** (*i*) (*b*), as

$$E_1 : E_2 : E_3 = 1 : 2 : 1$$

$$P(E_3) = \frac{1}{4}$$

(*ii*) (*b*), $P(E/E_2) = 4\%$, $E$ : Defective bulb.

(*iii*) (*d*), $E$: Defective bulb, $P(E) = \frac{1}{4} \times 0.04 + \frac{1}{2} \times 0.04 + \frac{1}{4} \times 0.05 = \frac{0.17}{4} = \frac{17}{400}$

(*iv*) (*d*), Using Baye's Theorem

$$P(E_2/E) = \frac{\frac{1}{2} \times 0.04}{\frac{17}{400}} = \frac{8}{17}$$

(*v*) (*a*), $E_1$ [Defective from $E_1$ is 0.01]

$$\text{Defective from } E_1 = \frac{1}{4} \times 0 \cdot 04 = 0 \cdot 01$$

$$\text{Defective from } E_2 = \frac{1}{2} \times 0 \cdot 04 = 0 \cdot 02$$

$$\text{Defective from } E_3 = \frac{1}{4} \times 0 \cdot 05 = 0 \cdot 0125$$

**49.** During the time of need and otherwise also people help the needy. It was found in survey that out of 200 people surveyed in a city 50 help the needy on regular basis, 120 contribute to Prime Minister relief fund and the rest help through NGO's. A person is selected who is in the need of a help, the probabilities of help through persons on regular basis, from Prime Minister relief fund and through NGO's are 0.15, 0.06 and 0.10 respectively.

Based on the above information answer the following.

(*i*) The probability of a help through NGO's is

(*a*) $\frac{1}{4}$ (*b*) $\frac{17}{20}$ (*c*) $\frac{5}{12}$ (*d*) $\frac{3}{20}$

(*ii*) The conditional probability of helping the needy through Prime Minister's relief fund is

(*a*) 0.15 (*b*) 0.06 (*c*) 0.10 (*d*) 0.69

(*iii*) The probability that the needy person received the help is

(*a*) 0.177 (*b*) 1.77 (*c*) 0.0885 (*d*) 0.67

(*iv*) The probability that needy person was helped through person on regular basis is

(*a*) $\frac{75}{177}$ (*b*) $\frac{72}{177}$ (*c*) $\frac{1}{59}$ (*d*) $\frac{6}{59}$

(*v*) In a city of population 1 lac, how many are expected to help on regular basis?

(*a*) 25000 (*b*) 60000 (*c*) 20000 (*d*) 5000

**Sol.** $A$ : helping on regular basis

$B$ : contributing to Prime Minister relief fund

$C$ : helping through NGO's

E : person needs help

(*i*) (*d*), $P(C) = 1 - [P(A) + P(B)] = 1 - \left[\frac{50}{200} + \frac{120}{200}\right] = \frac{3}{20}$

(*ii*) (*b*), P(E/B) = 0.06

(*iii*) (*c*),

$$P(E) = P(A) \cdot P(E/A) + P(B) \cdot P(E/B) + P(C) \cdot P(E/C)$$

$$= \frac{5}{20} \times 0.15 + \frac{12}{20} \times 0.06 + \frac{3}{20} \times 0.10$$

$$= \frac{0.75 + 0.72 + 0.30}{20} = \frac{1.77}{20} = 0.0885$$

(*iv*) (*a*), $P(A/E) = \frac{P(A) \cdot P(E/A)}{P(E)} = \frac{0.75}{1.77} = \frac{75}{177}$

(*v*) (*a*), Number of persons $= \frac{5}{20} \times 100000 = 25000$

# Part-II

## [Practice Papers]

# 1 PRACTICE PAPER

[Time Allowed: 90 minutes] [Maximum Marks: 40]

**General Instructions:**

1. The question paper contains three sections A, B and C. Each section is compulsory.
2. Section-A consists of 20 MCQs of 1 mark each. Any 16 questions are to be attempted.
3. Section-B consists of 20 MCQs of 1 mark each. Any 16 questions are to be attempted.
4. Section-C consists of 10 MCQs. Attempt any 8 questions.
5. There is no negative marking.
6. All questions carry equal marks.

## Section – A

**Section A consists of 20 questions of 1 mark each. Any 16 questions are to be attempted.**

**1.** If $\frac{d}{dx}f(x) = g(x)$, then antiderivative of $g(x)$ is

(*a*) $f(x)$ (*b*) $g(x)$ (*c*) $\frac{1}{2}[f(x)]^2$ (*d*) $\frac{1}{2}[g(x)]^2$

**Sol.** (*a*), as $\frac{d}{dx}f(x) = g(x) \Rightarrow \int g(x)\,dx = f(x)$.

**2.** The area of the region bounded by the curve $y = \frac{1}{x}$, $x$-axis and between $x = 1$, $x = 4$ is

(*a*) log 4 sq units (*b*) $\frac{1}{4}$ sq units (*c*) $\frac{1}{16}$ sq units (*d*) –log 4 sq units

**Sol.** (*a*), as curve is $y = \frac{1}{x}$, $x$-axis and between $x = 1$, $x = 4$

$$\text{Area} = \int_1^4 \frac{1}{x}\,dx = [\log|x|]_1^4$$

$$= \log 4 - \log 1 = \log 4 \text{ sq units.}$$

**3.** Integrating factor for the solution of differential equation $(y - x^3)dx + x\,dy = 0$ is

(*a*) $\frac{1}{y}$ (*b*) $\log y$ (*c*) $x$ (*d*) $x^2$

**Sol.** (*c*) Equation is $(y - x^3)\, dx + xdy = 0$

$$\Rightarrow \quad \frac{dy}{dx} = -\frac{y - x^3}{x} \quad \Rightarrow \quad \frac{dy}{dx} + \frac{1}{x}\cdot y = x^2$$

$$\text{Integrating factor} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$

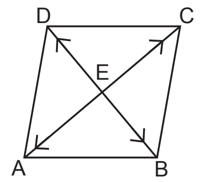
**4.** *ABCD* is a rhombus whose diagonals intersect at *E*. Then $\overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED}$ equals

(*a*) $\vec{0}$ (*b*) $\overrightarrow{AD}$ (*c*) $2\overrightarrow{BC}$ (*d*) $2\overrightarrow{AD}$

**Sol.** (*a*) as $\overrightarrow{EB} = -\overrightarrow{ED}$

and $\overrightarrow{EA} = -\overrightarrow{EC}$

$$\Rightarrow \quad \overrightarrow{EA} + \overrightarrow{EB} + \overrightarrow{EC} + \overrightarrow{ED} = \vec{0}$$




**5.** *P* is a point on the line segment joining the points (3, 5, –1) and (6, 3, –2). If *y*-coordinate of point *P* is 2, then its *x*-coordinate will be

(*a*) 2 (*b*) $\frac{17}{3}$ (*c*) $\frac{15}{2}$ (*d*) –5

**Sol.** (*c*) Let *P* divides the join of (3, 5, –1) and (6, 3, –2) in the ratio *k* : 1.

$$\therefore \quad \frac{3k+5}{k+1} = 2$$

$$\Rightarrow \quad 3k + 5 = 2k + 2 \Rightarrow k = -3$$

$\therefore$ *x*-coordinate is

$$\frac{6k+3}{k+1} = \frac{-18+3}{-3+1} = \frac{15}{2}$$

**6.** The value of *k*, for which the following distribution is a probability distribution

| X | 30 | 10 | –10 |
|---|---|---|---|
| P(X) | $\frac{1}{5}$ | $\frac{3}{10}$ | $k$ |

is

(*a*) $\frac{1}{3}$ (*b*) $\frac{1}{2}$ (*c*) $\frac{1}{10}$ (*d*) $\frac{1}{5}$

**Sol.** (*b*) as for probability distribution

$$\Sigma P(X) = 1 \quad \Rightarrow \quad \frac{1}{5} + \frac{3}{10} + k = 1$$

$$\Rightarrow \quad k = 1 - \frac{1}{2} = \frac{1}{2}.$$

**7.** Two events *A* and *B* are said to be independent if

(*a*) $P(A \cup B) = P(A)\cdot P(B)$ (*b*) $P(A \cap B) = 0$

(*c*) $P(A \cap B) = P(A)\cdot P(B)$ (*d*) none of these

**Sol.** (*c*) result

**8.** If $a$ is such that $\int_0^a x\,dx \le a + 4$, then

(*a*) $0 \le a \le 4$ (*b*) $-2 \le a \le 0$ (*c*) $a \le -2$ or $a \le 4$ (*d*) $-2 \le a \le 4$

**Sol.** (*d*) as $\int_0^a x\,dx \le a + 4 \Rightarrow \frac{a^2}{2} \le a + 4$

$\Rightarrow a^2 - 2a - 8 \le 0 \Rightarrow (a-1)^2 \le (3)^2$

$\Rightarrow -3 \le a - 1 \le 3 \Rightarrow -2 \le a \le 4$

**9.** The area enclosed by the curve $y = x^2$ and $y = 8$ is

(*a*) $\frac{64\sqrt{2}}{3}$ sq units (*b*) $\frac{512}{3}$ sq units (*c*) $\frac{64}{3}$ sq units (*d*) $24\sqrt{2}$ sq units

**Sol.** (*a*) as area $= 2\int_0^8 \sqrt{y}\,dy = 2\cdot\left[\frac{2}{3}y^{3/2}\right]_0^8 = \frac{4}{3}(8)^{3/2}$

$= \frac{4}{3}\cdot 8\cdot 2\sqrt{2}$ sq units $= \frac{64}{3}\sqrt{2}$ sq units

**10.** $F(x, y) = \frac{\sqrt{x^2 + y^2} + y}{x}$ is a homogeneous function of degree

(*a*) 0 (*b*) 1 (*c*) 2 (*d*) 3

**Sol.** (*a*) as if we find $F(\lambda x, \lambda y)$, we get $\lambda^{\circ} F(x, y)$

**11.** Mathematically a vector is defined as a

(*a*) line segment (*b*) directed line segment

(*c*) line (*d*) ray

**Sol.** (*b*)

**12.** If $\alpha, \beta, \gamma$ are the angles that a line makes with $x$, $y$ and $z$-axes respectively then the value of $\sin^2\alpha + \sin^2\beta + \sin^2\gamma$ is

(*a*) 1 (*b*) –1 (*c*) 2 (*d*) –2

**Sol.** (*c*) as $\alpha, \beta, \gamma$ are the angles which a line makes with $x$, $y$ and $z$ – axes

$\therefore \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$

$\Rightarrow 1 - \sin^2\alpha + 1 - \sin^2\beta + 1 - \sin^2\gamma = 1$

$\Rightarrow \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$

**13.** The vector normal to the plane $\vec{r}\cdot(3\hat{i} - 7\hat{k}) + 5 = 0$ is

(*a*) $\vec{r}$ (*b*) $3\hat{i} - 7\hat{j}$ (*c*) $3\hat{i} - 7\hat{k}$ (*d*) $3\hat{i} - 7\hat{k} + 5$

**Sol.** (*c*) as plane is $\vec{r}\cdot(3\hat{i} - 7\hat{k}) + 5 = 0$.

Normal vector is $\vec{n} = 3\hat{i} - 7\hat{k}$.

**14.** Given $P(A) = 0.4$, $P(B) = 0.7$ and $P(B/A) = 0.6$ then $P(A \cup B)$ is

(*a*) 1.1 (*b*) 0.86 (*c*) 0.46 (*d*) 0.16

**Sol.** (*b*) as $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$

$\Rightarrow \quad 0.6 \times 0.4 = P(A \cap B)$

$\Rightarrow \quad P(A \cap B) = 0.24$

Now, $P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.4 + 0.7 - 0.24 = 0.86$

**15.** If $\vec{a}$ and $\vec{b}$ are unit vectors, then what is the angle between $\vec{a}$ and $\vec{b}$ for $\sqrt{3}\,\vec{a} - \vec{b}$ to be a unit vector?

(*a*) 30° (*b*) 45° (*c*) 60° (*d*) 90°

**Sol.** (*a*) as

$$|\sqrt{3}\,\vec{a} - \vec{b}|^2 = \left(\sqrt{3}\,\vec{a} - \vec{b}\right)^2$$

$$= 3\vec{a}^2 + \vec{b}^2 - 2\sqrt{3}\,\vec{a}\cdot\vec{b}$$

$$\Rightarrow \quad 1 = 3 + 1 - 2\sqrt{3}\;\vec{a}\cdot\vec{b}$$

$$\Rightarrow \quad \vec{a}\cdot\vec{b} = \frac{\sqrt{3}}{2}$$

$$\therefore \quad \cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}||\vec{b}|} = \frac{\sqrt{3}}{2} \Rightarrow \theta = 30°$$

**16.** $\displaystyle\int \frac{\sin x + \cos x}{\sqrt{1+\sin 2x}}dx$, $\dfrac{3\pi}{4} < x < \dfrac{7\pi}{4}$ is equal to

(*a*) $\log|\sin x + \cos x|$ (*b*) $x$ (*c*) $\log|x|$ (*d*) $-x$

**Sol.** (*d*) as

$$\int \frac{\sin x + \cos x}{\sqrt{\sin^2 x + \cos^2 x + 2\sin x\cos x}}dx = \int \frac{\sin x + \cos x}{|\sin x + \cos x|}dx,$$

$$\Rightarrow \quad -\int 1\cdot dx = -x + C \qquad \left\{\text{as } \sin x + \cos x < 0 \text{ for } \frac{3\pi}{4} < x < \frac{7\pi}{4}\right\}$$

**17.** $\displaystyle\int_0^{\frac{\pi}{2}} \frac{dx}{1+\sin x}$ equals to

(*a*) 0 (*b*) $\dfrac{1}{2}$ (*c*) 1 (*d*) $\dfrac{3}{2}$

**Sol.** (*c*) as

$$\int_0^{\frac{\pi}{2}} \frac{dx}{1+\cos\left(\frac{\pi}{2}-x\right)} = \int_0^{\frac{\pi}{2}} \frac{1}{2}\sec^2\left(\frac{\pi}{4}-\frac{x}{2}\right)dx$$

$$= \frac{1}{2}\cdot\left[\frac{\tan\left(\frac{\pi}{4}-\frac{x}{2}\right)}{-\frac{1}{2}}\right]_0^{\frac{\pi}{2}}$$

$$= -\tan\left(\frac{\pi}{4}-\frac{\pi}{4}\right) + \tan\left(\frac{\pi}{4}-0\right) = 1.$$

**18.** The particular solution of the differential equation $\cos x\dfrac{dy}{dx} + y = \sin x$, given that $y = 2$ when $x = 0$ is

(*a*) $(\sec x + \tan x)y = \sec x + \tan x - x + 1$ (*b*) $(\sec x - \tan x)y = \sec x + \tan x + x - 1$

(*c*) $(\text{cosec } x + \tan x)y = \text{cosec } x + \tan x + x - 1$ (*d*) $(\sec^2 x - \tan^2 x)y = \sec x + \tan x - x + 1$

**Sol.** (*a*) $\dfrac{dy}{dx} + \sec x \cdot y = \tan x,$

$$\text{I.F.} = e^{\int \sec x\, dx} = e^{\log|\sec x + \tan x|} = \sec x + \tan x$$

Solution is $(\sec x + \tan x)y = \int (\sec x + \tan x) \tan x\, dx$

$\Rightarrow$ $(\sec x + \tan x)y = \int (\sec x \tan x + \sec^2 x - 1)\, dx$

$\Rightarrow$ $(\sec x + \tan x)y = \sec x + \tan x - x + C$ ...(*i*)

When $x = 0$, $y = 2$

$\Rightarrow$ $(\sec 0 + \tan 0)2 = \sec 0 + \tan 0 - 0 + C$

$\Rightarrow$ $2 = 1 + C \Rightarrow C = 1$

Substituting in (*i*) we get

$(\sec x + \tan x)y = \sec x + \tan x - x + 1$ is the required solution.

**19.** Given vector $\overrightarrow{PQ} = 2\hat{i} + \hat{j} - 3\hat{k}$ and position vector of point $P$ is $3\hat{j} - 2\hat{k}$, then position vector of point $Q$ is

(*a*) $2\hat{i} - 2\hat{j} - \hat{k}$ (*b*) $2\hat{i} + 4\hat{j} - 5\hat{k}$ (*c*) $2\hat{i} + 4\hat{j} - \hat{k}$ (*d*) $2\hat{i} - 2\hat{j} - 5\hat{k}$

**Sol.** (*b*) as $\overrightarrow{PQ}$ = position vector of $Q$ – position vector of $P$

$\Rightarrow$ Position vector of Q $= 2\hat{i} + \hat{j} - 3\hat{k} + 3\hat{j} - 2\hat{k} = 2\hat{i} + 4\hat{j} - 5\hat{k}$

**20.** Equation of the perpendicular drawn from the point with position vector $2\hat{i} - \hat{j} + \hat{k}$ to the plane $\vec{r} \cdot (\hat{i} - 3\hat{k}) = 5$ is

(*a*) $\vec{r} = \lambda(\hat{i} - 3\hat{k})$ (*b*) $\vec{r} = \lambda(2\hat{i} - \hat{j} + \hat{k})$

(*c*) $\vec{r} = (2\hat{i} - \hat{j} + \hat{k}) + \lambda(\hat{i} - 3\hat{k})$ (*d*) $\vec{r} = (\hat{i} - 3\hat{j}) + \lambda(2\hat{i} - \hat{j} + \hat{k})$

**Sol.** (*c*), as $\vec{a} = 2\hat{i} - \hat{j} + \hat{k}$ and direction of line is along $\vec{n} = \hat{i} - 3\hat{k}$.

## Section – B

**Section B consists of 20 questions of 1 mark each. Any 16 questions are to be attempted.**

**21.** Bag $A$ contains 3 red and 5 black balls and bag $B$ contains 2 red and 4 black balls. A ball is drawn from one of the bags. The probability that ball drawn is red is

(*a*) $\dfrac{17}{24}$ (*b*) $\dfrac{17}{48}$ (*c*) $\dfrac{3}{8}$ (*d*) $\dfrac{1}{3}$

**Sol.** (*b*) as

$$P\text{ (red)} = P(A) \cdot P(R/A) + P(B) \cdot P(R/B)$$
$$= \frac{1}{2}\cdot\frac{3}{8}+\frac{1}{2}\cdot\frac{2}{6} = \frac{3}{16}+\frac{1}{6}=\frac{17}{48}.$$

**22.** The value of integral $\int_{-\frac{1}{2}}^{\frac{1}{2}} \cos x.\log\left(\frac{1+x}{1-x}\right)dx$ is

(*a*) 0 (*b*) $\frac{1}{2}$ (*c*) $\frac{3}{2}$ (*d*) none of these

**Sol.** (*a*) as function is odd function.

**23.** For which value of *p*, is $(\hat{i}+\hat{j}+\hat{k})p$ a unit vector?

(*a*) $\pm\frac{1}{\sqrt{3}}$ (*b*) $\pm\sqrt{3}$ (*c*) $\pm 1$ (*d*) $\pm\frac{1}{3}$

**Sol.** (*a*) as for a unit vector, $|p\hat{i}+p\hat{j}+p\hat{k}| = 1$ $\Rightarrow$ $\sqrt{p^2+p^2+p^2}=1$

$$\Rightarrow \quad p = \pm\frac{1}{\sqrt{3}}$$

**24.** Let *A* and *B* be two given independent events such that $P(A) = p$ and $P(B) = q$ and *P*(exactly one of *A*, *B*) = $\frac{2}{3}$, then value of $3p + 3q - 6pq$ is

(*a*) 2 (*b*) –2 (*c*) 4 (*d*) –4

**Sol.** (*a*) as $P(A)\cdot P(\overline{B}) + P(\overline{A})\cdot P(B) = \frac{2}{3}$

$$\Rightarrow \quad p\cdot(1-q)+(1-p)q = \frac{2}{3}$$
$$\Rightarrow \quad p - pq + q - pq = \frac{2}{3}$$
$$\Rightarrow \quad 3p + 3q - 6pq = 2$$

**25.** $\int \frac{1}{x^2(x^4+1)^{3/4}}\,dx$ is equal to

(*a*) $-\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$ (*b*) $(x^4+1)^{\frac{1}{4}}+C$ (*c*) $\left(1-\frac{1}{x^4}\right)^{\frac{1}{4}}+C$ (*d*) $-\left(1+\frac{1}{x^4}\right)^{\frac{3}{4}}+C$

**Sol.** (*a*) as

$$\int \frac{1}{x^2\cdot x^3\left(1+\frac{1}{x^4}\right)^{3/4}}dx = \int\frac{1}{x^5\left(1+\frac{1}{x^4}\right)^{3/4}}dx$$

Let $1+\frac{1}{x^4}=t$ $\Rightarrow$ $\frac{dx}{x^5} = -\frac{dt}{4}$

$$= \int -\frac{dt}{4t^{3/4}} = -\frac{1}{4}[4t^{1/4}]+C$$
$$= -\left(1+\frac{1}{x^4}\right)^{\frac{1}{4}}+C$$

**26.** The area enclosed by the circle $x^2 + y^2 = 8$ is

(*a*) $16\pi$ sq units (*b*) $2\sqrt{2}\,\pi$ sq units (*c*) $8\pi^2$ sq units (*d*) $8\pi$ sq units

**Sol.** (*d*) as for circle $x^2 + y^2 = 8$, centre is (0, 0), radius = $\sqrt{8}$.

$$\therefore \qquad \text{Area} = 4\int_0^{2\sqrt{2}} \sqrt{8 - x^2}\, dx = 8\pi \text{ sq units}$$

$$\left\{\text{Use } \int\sqrt{a^2 - x^2}\, dx = \frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2}\sin^{-1}\frac{x}{a} + C\right\}$$

**27.** If $p$ and $q$ are the degree and order of the differential equation $\left(\dfrac{d^2y}{dx^2}\right)^2 + 3\dfrac{dy}{dx} + \dfrac{d^3y}{dx^3} = 4$, then the value of $2p - 3q$ is

(*a*) 7 (*b*) –7 (*c*) 3 (*d*) –3

**Sol.** (*b*) as degree $p = 1$ and order $q = 3$

$$\therefore \qquad 2p - 3q = 2 - 9 = -7$$

**28.** Area of parallelogram, whose diagonals are along vectors $\hat{i} + 2\hat{k}$ and $2\hat{j} - 3\hat{k}$ is

(*a*) $\sqrt{29}$ (*b*) $-4\hat{i} + 3\hat{j} + 2\hat{k}$ (*c*) $\frac{1}{2}\sqrt{29}$ (*d*) none of these

**Sol.** (*c*), as area $= \frac{1}{2}|(\hat{i} + 2\hat{k}) \times (2\hat{j} - 3\hat{k})|$

$$(\hat{i} \times 2\hat{k}) \times (2\hat{j} - 3\hat{k}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 0 & 2 \\ 0 & 2 & -3 \end{vmatrix}$$

$$= -4\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\text{Area} = \frac{1}{2}\left|\sqrt{(-4)^2 + 3^2 + 2^2}\right|$$

$$= \frac{1}{2} \times \sqrt{29}$$

**29.** Distance of plane $\vec{r} \cdot (2\hat{i} + 3\hat{j} - 6\hat{k}) + 2 = 0$, from origin is

(*a*) 2 (*b*) 14 (*c*) $\frac{2}{7}$ (*d*) $-\frac{2}{7}$

**Sol.** (*c*) as normal form of plane is

$$\vec{r} \cdot \left(-\frac{2}{7}\hat{i} - \frac{3}{7}\hat{j} + \frac{6}{7}\hat{k}\right) = \frac{2}{7}$$

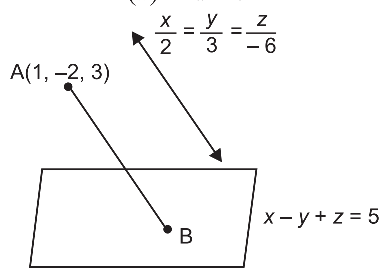
$$\therefore \qquad p = \frac{2}{7}$$

$$\therefore \qquad \text{distance} = \frac{2}{7}$$

**30.** The distance of the point (1, –2, 3) from the plane $x - y + z = 5$ measured parallel to the line $\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}$ is.

(*a*) 1 unit (*b*) 4 units (*c*) 3 units (*d*) 2 units

**Sol.** (*a*) Let $AB$ is parallel to the given line




$\therefore$ equation is $\frac{x-1}{2} = \frac{y+2}{3} = \frac{z-3}{-6}$

General point on the line is

$B(2\lambda + 1, 3\lambda - 2, -6\lambda + 3)$ ...(*i*)

If this point lies on the plane then

$2\lambda + 1 - 3\lambda + 2 - 6\lambda + 3 = 5 \Rightarrow \lambda = \frac{1}{7}$

Substituting in (*i*), we get point of intersection $B\left(\frac{2}{7}+1, \frac{3}{7}-2, \frac{-6}{7}+3\right)$ i.e. $B\left(\frac{9}{7}, \frac{-11}{7}, \frac{15}{7}\right)$

$\therefore$ distance,

$$AB = \sqrt{\left(\frac{9}{7}-1\right)^2 + \left(\frac{-11}{7}+2\right)^2 + \left(\frac{15}{7}-3\right)^2} = \sqrt{\frac{4}{49}+\frac{9}{49}+\frac{36}{49}} = \sqrt{1} = 1 \text{ unit}$$

**31.** $\int \cos^3 x . e^{\log(\sin x)}\, dx$ is equal to

(*a*) $-\frac{\cos^4 x}{4} + C$ (*b*) $-\frac{\sin^4 x}{4} + C$ (*c*) $\frac{e^{\sin x}}{4} + C$ (*d*) none of these

**Sol.** (*a*) Let $I = \int \cos^3 x e^{\log(\sin x)} dx$ |Let $\cos x = t \Rightarrow -\sin x dx = dt$

$$= \int \cos^3 x \sin x \, dx$$

$$= \int -t^3 dt = -\frac{t^4}{4} + C = -\frac{\cos^4 x}{4} + C$$

**32.** General solution of the differential equation $\log\left(\frac{dy}{dx}\right) = 2x + y$ is

(*a*) $e^{-y} = \frac{1}{2}e^{2x} + C$ (*b*) $\frac{1}{e^y} + \frac{1}{2}e^{2x} + C$ (*c*) $-e^{-y} = \frac{1}{2}e^{2x} + C$ (*d*) $e^y = \frac{1}{2}e^{2x} + C$

**Sol.** (*c*) as $\frac{dy}{dx} = e^{2x+y} = e^{2x} \cdot e^y$

$\Rightarrow$ $\int e^{-y} dy = \int e^{2x} dx$

$\Rightarrow$ $-e^{-y} = \frac{1}{2}e^{2x} + C$

**33.** If $x = \int_0^y \frac{dt}{\sqrt{1+9t^2}}$ and $\frac{d^2y}{dx^2} = ay$, then value of $a$ is equal to

(*a*) 3 (*b*) 6 (*c*) 9 (*d*) 1

**Sol.** (*c*), as $\frac{dx}{dy} = \frac{1}{\sqrt{1+9y^2}} \Rightarrow \frac{dy}{dx} = \sqrt{1+9y^2}$

$\Rightarrow \frac{d^2y}{dx^2} = \frac{18y}{2\sqrt{1+9y^2}} \cdot \frac{dy}{dx} \Rightarrow \frac{d^2y}{dx^2} = 9y \cdot \frac{1}{\sqrt{1+9y^2}} \cdot \sqrt{1+9y^2} = 9y$

Hence, $a = 9$

**34.** If for non zero vectors $\vec{a}$ and $\vec{b}$, $\vec{a} \times \vec{b}$ is a unit vector and $|\vec{a}| = |\vec{b}| = \sqrt{2}$, then angle θ between vectors $\vec{a}$ and $\vec{b}$ is

(*a*) $\frac{\pi}{2}$ (*b*) $\frac{\pi}{3}$ (*c*) $\frac{\pi}{6}$ (*d*) $-\frac{\pi}{2}$

**Sol.** (*c*) as $\sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{6}$

**35.** If $|\vec{a}| = 5$, $|\vec{b}| = 13$ and $|\vec{a} \times \vec{b}| = 25$, then $\vec{a} \cdot \vec{b}$ is equal to

(*a*) 12 (*b*) 5 (*c*) 13 (*d*) 60

**Sol.** (*d*) as $|\vec{a} \times \vec{b}|^2 + (\vec{a} \cdot \vec{b})^2 = |\vec{a}|^2 |\vec{b}|^2$

$\Rightarrow 25^2 + (\vec{a} \cdot \vec{b})^2 = 5^2 \times 13^2$

$\Rightarrow (\vec{a} \cdot \vec{b})^2 = 25 \times 169 - 25 \times 25$

$= 25(169 - 25) = 25 \times 144$

$\Rightarrow \vec{a} \cdot \vec{b} = 60$

**36.** Direction cosines of a unit vector perpendicular to the plane $\vec{r} \cdot (6\hat{i} - 3\hat{j} - 2\hat{k}) - 1 = 0$ are

(*a*) 6, –3, –2 (*b*) $\frac{6}{7}, -\frac{3}{7}, -\frac{2}{7}$ (*c*) $-\frac{6}{7}, \frac{3}{7}, -\frac{2}{7}$ (*d*) none of these

**Sol.** (*b*) as plane is $\vec{r} \cdot (6\hat{i} - 3\hat{j} - 2\hat{k}) = 1 \Rightarrow \vec{r} \cdot \left(\frac{6}{7}\hat{i} - \frac{3}{7}\hat{j} - \frac{2}{7}\hat{k}\right) = \frac{1}{7}$

**37.** The value of $\int_0^2 x[x]\, dx$ is

(*a*) $\frac{7}{2}$ (*b*) $\frac{3}{2}$ (*c*) $\frac{5}{2}$ (*d*) none of these

**Sol.** (*b*)

**38.** Degree of differential equation $t^2\frac{d^2s}{dt^2} - st\left(\frac{ds}{dt}\right)^2 = 5$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) none of these

**Sol.** (*a*), as exponent of the highest order derivative is 1.

**39.** $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta} dx$ is equal to

(*a*) $2(\sin x + x\cos\theta) + C$ (*b*) $2(\sin x - x\cos\theta) + C$

(*c*) $2(\sin x + 2x\cos\theta) + C$ (*d*) $2(\sin x - 2x\cos\theta) + C$

**Sol.** (*a*) as $\int \frac{\cos 2x - \cos 2\theta}{\cos x - \cos\theta} dx = \int \frac{2(\cos^2 x - \cos^2\theta)}{\cos x - \cos\theta} dx$

$= 2\int (\cos x + \cos\theta)\, dx$

$= 2\sin x + 2x \cdot \cos\theta + C = 2(\sin x + x\cos\theta) + C$

**40.** Area of the region bounded by the curve $x = 2y + 3$, the $y$-axis and between $y = -1$ and $y = 1$ is

(*a*) 4 sq units (*b*) $\frac{3}{2}$ sq units (*c*) 6 sq units (*d*) 8 sq units

**Sol.** (*c*), as area $= \int_{-1}^{1} (2y+3)\,dy = \left[2\frac{y^2}{2} + 3y\right]_{-1}^{1} = \left(\frac{1}{2}+3\right) - \left(\frac{1}{2} - 3\right) = 6$ sq units

## Section – C

**Section C consists of 10 questions. Attempt any 8 questions.**

**Questions 46-50 are based on a case-study.**

**41.** The area of a parallelogram whose one diagonal is $2\hat{i} + \hat{j} - 2\hat{k}$ and one side is $3\hat{i} + \hat{j} - \hat{k}$ is

(*a*) $\hat{i} - 4\hat{j} - \hat{k}$ (*b*) $3\sqrt{2}$ sq units (*c*) $6\sqrt{2}$ sq units (*d*) 6 sq units

**Sol.** (*b*) as area of parallelogram $= \left\| \begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -2 \\ 3 & 1 & -1 \end{matrix} \right\|$

$= |\hat{i} - 4\hat{j} - \hat{k}| = \sqrt{1+16+1} = 3\sqrt{2}$ sq units

**42.** The particular solution of the differential equation $\frac{dy}{dx} = y \tan x$, given that $y = 1$ when $x = 0$ is

(*a*) $y = \cos x$ (*b*) $y = \sec x$ (*c*) $y = \tan x$ (*d*) $y = \sec x \tan x$

**Sol.** (*b*) as $\int \frac{dy}{y} = \int \tan x\, dx$

$\Rightarrow$ $\log |y| = \log|\sec x| + \log C$

$\Rightarrow$ $y = C \sec x$ ...(*i*)

Given $y = 1, x = 0 \Rightarrow 1 = C \sec 0 \Rightarrow C = 1$

$\therefore$ solution is $y = \sec x$ [from (*i*)]

**43.** The line joining the points (0, 5, 4) and (1, 3, 6) meets $XY$-plane at the point

(*a*) (–2, 9, 0) (*b*) (4, –3, 0) (*c*) (1, –2, 0) (*d*) (1, 3, 0)

**Sol.** (*a*) as line is $\frac{x-1}{1} = \frac{y-3}{-2} = \frac{z-6}{2} = \lambda$ (say)

General point on the line is $(\lambda + 1, -2\lambda + 3, 2\lambda + 6)$

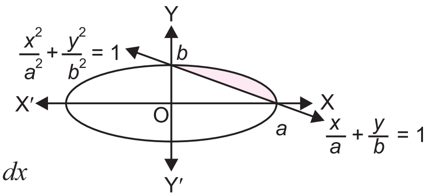
If it meets $XY$-plane, then $2\lambda + 6 = 0 \Rightarrow \lambda = -3$

$\therefore$ Point is (–2, 9, 0)

**44.** The area of the smaller region between the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ and the line $\frac{x}{a}+\frac{y}{b}=1$ in first quadrant is

(*a*) $\frac{1}{2}ab$ sq units (*b*) $\frac{1}{2}\pi ab$ sq units (*c*) $\pi ab$ sq units (*d*) $\frac{ab}{4}(\pi-2)$ sq units

**Sol.** (*d*) $y_1: \frac{x^2}{a^2}+\frac{y^2}{b^2}=1;\ y_2=\frac{x}{a}+\frac{y}{b}=1$




$$\text{Area} = \int_0^a (y_1-y_2)\,dx$$

$$= \int_0^a \left\{\frac{b}{a}\sqrt{a^2-x^2}-\frac{b}{a}(a-x)\right\}dx$$

$$= \frac{b}{a}\left[\frac{x}{2}\sqrt{a^2-x^2}+\frac{a^2}{2}\sin^{-1}\frac{x}{a}-ax+\frac{x^2}{2}\right]_0^a$$

$$= \frac{b}{a}\left[\left(0+\frac{a^2}{2}\cdot\sin^{-1}1-a^2+\frac{a^2}{2}\right)-0\right]$$

$$= \frac{b}{a}\left[\frac{a^2}{2}\cdot\frac{\pi}{2}-\frac{a^2}{2}\right]=\frac{b}{a}\cdot\frac{a^2}{2}\left(\frac{\pi}{2}-1\right)$$

$$= \frac{ab}{4}(\pi-2)\text{ sq units}$$

**45.** The equation of the line passing through the point $P(4, 6, 2)$ and the point of intersection of the line $\frac{x-1}{3}=\frac{y}{2}=\frac{z+1}{7}$ and the plane $x+y-z=8$.

(*a*) $\frac{x-4}{1}=\frac{y-6}{1}=\frac{z-2}{1}$ (*b*) $\frac{x-6}{1}=\frac{y-4}{1}=\frac{z-2}{2}$

(*c*) $\frac{x-2}{1}=\frac{y-2}{1}=\frac{z-4}{2}$ (*d*) $\frac{x-4}{2}=\frac{y-6}{2}=\frac{z-2}{1}$

**Sol.** (*a*) General point on the line $\frac{x-1}{3}=\frac{y}{2}=\frac{z+1}{7}$ is

$Q\,(3\lambda+1, 2\lambda, 7\lambda-1)$. ...(*i*)

If this point lies on the plane then

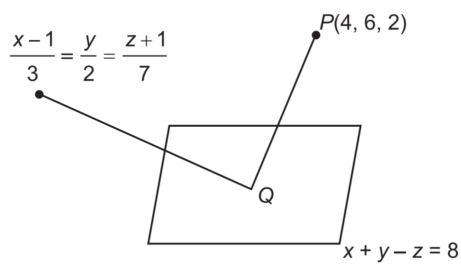
$3\lambda+1+2\lambda-7\lambda+1=8$

$\Rightarrow\ -2\lambda=6\ \Rightarrow\ \lambda=-3$

Substituting in (*i*), point of intersection is

$Q\,(-8, -6, -22)$

Direction ratios of $PQ$ are 12, 12, 24 or 1, 1, 2.




$\therefore$ equation of $PQ$ is $\frac{x-4}{1}=\frac{y-6}{1}=\frac{z-2}{2}$.

## Case-Based Questions

A coach is training 3 players. He observes that the player A can hit a target 4 times in 5 shots, player B can hit 3 times in 4 shots and the player C can hit 2 times in 3 shots

From this situation answer the following:

**46.** Let the target is hit by $A$, $B$: the target is hit by $B$ and, $C$: the target is hit by $A$ and $C$. Then, the probability that $A$, $B$ and, $C$ all will hit, is

(*a*) $\frac{4}{5}$ (*b*) $\frac{3}{5}$ (*c*) $\frac{2}{5}$ (*d*) $\frac{1}{5}$

**Sol.** (*c*) $\frac{2}{5}$

**47.** Referring to Q. 46, what is the probability that $B$, $C$ will hit and $A$ will lose?

(*a*) $\frac{1}{10}$ (*b*) $\frac{3}{10}$ (*c*) $\frac{7}{10}$ (*d*) $\frac{4}{10}$

**Sol.** (*a*) $\frac{1}{10}$

**48.** With reference to the events mentioned in Q46, what is the probability that 'any two of $A$, $B$ and $C$ will hit?

(*a*) $\frac{1}{30}$ (*b*) $\frac{11}{30}$ (*c*) $\frac{17}{30}$ (*d*) $\frac{13}{30}$

**Sol.** (*d*) $\frac{13}{30}$

**49.** What is the probability that 'none of them will hit the target'?

(*a*) $\frac{1}{30}$ (*b*) $\frac{1}{60}$ (*c*) $\frac{1}{15}$ (*d*) $\frac{2}{15}$

**Sol.** (*b*) $\frac{1}{60}$

**50.** What is the probability that at least one of $A$, $B$ or $C$ will hit the target?

(*a*) $\frac{59}{60}$ (*ii*) $\frac{2}{5}$ (*iii*) $\frac{3}{5}$ (*iv*) $\frac{1}{60}$

**Sol.** (*a*) $\frac{59}{60}$

# 2 | PRACTICE PAPER

[Time Allowed: 90 minutes] [Maximum Marks: 40]

**General Instructions:** As given in Practice Paper – 1.

## Section – A

**Section A consists of 20 questions of 1 mark each. Any 16 questions are to be attempted.**

**1.** $\int \cot^2 x\, dx$ equals to

(*a*) $\cot x - x + C$ (*b*) $\cot x + x + C$ (*c*) $-\cot x + x + C$ (*d*) $-\cot x - x + C$

**Sol.** (*d*) $\int (\text{cosec}^2 x - 1) dx = -\cot x - x + C$

**2.** Order of differential equation corresponding to family of curves $y = Ae^{2x} + Be^{-2x}$ is

(*a*) 2 (*b*) 1 (*c*) 3 (*d*) 4

**Sol.** (*a*) as there are two arbitrary constants and we have to differentiate twice.

**3.** If $|\vec{a}| = \sqrt{3}$, $|\vec{b}| = 2$ and angle between $\vec{a}$ and $\vec{b}$ is 60°, then $\vec{a} . \vec{b}$ is

(*a*) $\sqrt{3}$ (*b*) 2 (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{\sqrt{3}}$

**Sol.** (*a*) as $\vec{a} . \vec{b} = |\vec{a}||\vec{b}| \cos 60° = \sqrt{3} \times 2 \times \frac{1}{2} = \sqrt{3}$.

**4.** A line makes angle α, β, γ with *x*-axis, *y*-axis and *z*-axis respectively then cos 2α + cos 2β + cos 2γ is equal to

(*a*) 2 (*b*) 1 (*c*) –2 (*d*) –1

**Sol.** (*d*) as $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

$$\Rightarrow \quad \frac{1 + \cos 2\alpha}{2} + \frac{1 + \cos 2\beta}{2} + \frac{1 + \cos 2\gamma}{2} = 1$$

$$\Rightarrow \quad \cos 2\alpha + \cos 2\beta + \cos 2\gamma = -1$$

**5.** Let *A* and *B* be two given mutually exclusive events. Then *P*(*A*/*B*) is

(*a*) 1 (*b*) $\frac{P(A \cup B)}{P(B)}$ (*c*) $\frac{P(A \cup B)}{P(B)}$ (*d*) 0

**Sol.** (*d*) as $P(A \cap B) = 0$ and $P(A/B) = \frac{P(A \cap B)}{P(B)}$.

**6.** The probability of $A$, $B$ and $C$ solving a problem are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. Then the probability that the problem will be solved is

(*a*) $\frac{1}{2}$ (*b*) $\frac{3}{4}$ (*c*) $\frac{1}{4}$ (*d*) none of these

**Sol.** (*b*) as $P$(problem solved)

$$= 1 - P(\text{none solves})$$

$$= 1 - \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = 1 - \frac{1}{4} = \frac{3}{4}$$

**7.** The value of integral $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx$ is

(*a*) $\log 2$ (*b*) $\frac{1}{20} \log 2$ (*c*) $\frac{1}{20} \log 3$ (*d*) $\log 5$

**Sol.** (*c*), as $\int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{9 + 16 \sin 2x} dx = \int_0^{\frac{\pi}{4}} \frac{\sin x + \cos x}{25 - 16 (\cos x - \sin x)} dx$

Let $\cos x - \sin x = t$
$\Rightarrow -(\sin x + \cos x)dx = dt$
When $x = \frac{\pi}{4}$ $\Rightarrow$ $t = 0$
and when $x = 0$, $\Rightarrow$ $t = 1$

$$= -\int_1^0 \frac{dt}{25 - 16t}$$

$$= \int_0^1 \frac{dt}{\frac{25}{16} - t} = \frac{1}{16} \int_0^1 \frac{dt}{\left(\frac{5}{4}\right)^2 - (\sqrt{t})^2}$$

$$= \frac{1}{16} \left[ \frac{1}{2 \times \frac{5}{4}} \log \left| \frac{\frac{5}{4} + \sqrt{t}}{\frac{5}{4} - \sqrt{t}} \right| \right]_0^1$$

$$= \frac{1}{40} \log 9 = \frac{1}{20} \log 3.$$

**8.** The degree of the differential equation $\frac{d^2y}{dx^2} + 3\left(\frac{dy}{dx}\right)^2 = x^2 \log\left(\frac{d^2y}{dx^2}\right)$ is

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) not defined

**Sol.** (*d*) as equation cannot be represented as a polynomial of derivatives.

**9.** A vector equally inclined to axes is

(*a*) $\hat{i} + \hat{j} + \hat{k}$ (*b*) $\hat{i} - \hat{j} + \hat{k}$ (*c*) $\hat{i} - \hat{j} - \hat{k}$ (*d*) $-\hat{i} + \hat{j} - \hat{k}$

**Sol.** (*a*) as direction ratios are 1, 1, 1 and direction cosines $\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} \Rightarrow \cos \alpha = \cos \beta = \cos \gamma$
$\Rightarrow \alpha = \beta = \gamma$

**10.** If the direction cosines of a given line are $\frac{1}{k}, \frac{1}{k}, \frac{1}{k}$ then the value of $k$ is

(*a*) $\frac{1}{\sqrt{2}}$ (*b*) $\pm \frac{1}{\sqrt{3}}$ (*c*) 1 (*d*) $\pm \sqrt{3}$

**Sol.** (*d*) as $\frac{1}{k}, \frac{1}{k}, \frac{1}{k}$ are direction cosines of a line

$$\therefore \quad \frac{1}{k^2} + \frac{1}{k^2} + \frac{1}{k^2} = 1 \quad \Rightarrow \quad \frac{3}{k^2} = 1$$

$$\Rightarrow \quad k^2 = 3 \quad \Rightarrow \quad k = \pm\sqrt{3}$$

**11.** Integrating factor for the solution of differential equation $\frac{dy}{dx} + 2y \tan x = \sin x$ is

(*a*) $\sec^2 x$ (*b*) $\sec x$ (*c*) $\log |\sec x|$ (*d*) $\tan x$

**Sol.** (*a*) Equation is $\frac{dy}{dx} + (2 \tan x) \cdot y = \sin x$

$$\text{Integrating factor} = e^{\int 2 \tan x\, dx} = e^{2\int \tan x\, dx} = e^{2 \log |\sec x|}$$

$$= \sec^2 x$$

**12.** $\int \frac{\sqrt{\tan x}}{\sin x . \cos x}\, dx$ is equal to

(*a*) $2\sqrt{\cot x} + C$ (*b*) $\frac{\sqrt{\tan x}}{2} + C$ (*c*) $2\sqrt{\tan x} + C$ (*d*) none of these

**Sol.** (*c*), $\int \frac{\sqrt{\tan x}}{\sin x \cos x}\, dx = \int \frac{\sqrt{\tan x}}{\frac{\sin x}{\cos x} . \cos^2 x}\, dx$

$$= \int \frac{\sqrt{\tan x}}{\tan x} . \sec^2 x\, dx = \int \frac{1}{\sqrt{\tan x}} dx$$

Let $\tan x = t$
$\Rightarrow \sec^2 x\, dx = dt$

$$= \int \frac{1}{\sqrt{t}}\, dt = 2\sqrt{t} + C$$

**13.** If $|\vec{a}| = 4$ and $-3 \le \lambda \le 2$ then the range of $|\lambda \vec{a}|$ is

(*a*) [0, 8] (*b*) [–12, 8] (*c*) [0, 12] (*d*) [8, 12]

**Sol.** (*c*) as $|\lambda \vec{a}| = |\lambda||\vec{a}| = 4|\lambda|$

Also $-3 \le \lambda \le 2 \Rightarrow 0 \le |\lambda| \le 3 \quad \Rightarrow \quad 0 \le 4|\lambda| \le 12$

**14.** Let $A$ and $B$ be two given events such that $P(A) = 0.6$, $P(B) = 0.2$ and $P(A/B) = 0.5$. Then $P(A'/B')$ is

(*a*) $\frac{1}{10}$ (*b*) $\frac{3}{10}$ (*c*) $\frac{3}{8}$ (*d*) $\frac{6}{7}$

**Sol.** (*c*) as $P(A/B) = \frac{P(A \cap B)}{P(B)}$

$$\Rightarrow \quad P(A \cap B) = 0.5 \times 0.2 = 0.1$$

$$P(A'/B') = \frac{P(A' \cap B')}{P(B')} = \frac{1 - P(A \cup B)}{1 - P(B)} = \frac{3}{8}$$

**15.** The value of $k$, for which the following distribution is a probability distribution

| $X$ | 30 | 10 | –10 |
|---|---|---|---|
| $P(X)$ | $\frac{1}{20}$ | $\frac{3}{10}$ | $k$ |

is

(*a*) $\frac{1}{3}$ (*b*) $\frac{13}{20}$ (*c*) $\frac{1}{10}$ (*d*) $\frac{1}{5}$

**Sol.** (*b*) as for probability distribution

$$\Sigma P(X) = 1$$

$$\Rightarrow \quad \frac{1}{20} + \frac{3}{10} + k = 1$$

$$\Rightarrow \quad k = 1 - \frac{3}{10} - \frac{1}{20}$$

$$\Rightarrow \quad k = \frac{13}{20}$$

**16.** Given vector $\vec{a}$, then $-2\vec{a}$ is a vector whose

(*a*) magnitude is twice that of $\vec{a}$ and direction is same as that of $\vec{a}$

(*b*) magnitude is twice that of $\vec{a}$ and direction is opposite to that of $\vec{a}$

(*c*) magnitude is same as that of $\vec{a}$ and direction is opposite to that of $\vec{a}$

(*d*) none of these

**Sol.** (*b*), result related to $\vec{a}$ and $k\vec{a}$, $k$ is scalar.

**17.** $\int \sec^2(7-x)\,dx$ is equal to

(*a*) $\tan(7-x) + C$ (*b*) $2\sec^2(7-x)\tan x + C$

(*c*) $\sec^3(7-x) + C$ (*d*) $-\tan(7-x) + C$

**Sol.** (*d*), as $\int \sec^2(7-x)\,dx = -\int \sec^2 t\,dt$

$$= -\tan t + C = -\tan(7-x) + C$$

Let $7 - x = t$
$\Rightarrow -dx = dt$

**18.** $\int_{-1}^{1} |(1-x)|\,dx$ is equal to

(*a*) $\frac{1}{2}$ (*b*) –1 (*c*) 2 (*d*) 1

**Sol.** (*c*), as $\int_{-1}^{1} |1-x|\,dx = \int_{-1}^{1} (1-x)\,dx$, $1 - x \geq 0$ for $-1 \leq x \leq 1$

$$= \left[x - \frac{x^2}{2}\right]_{-1}^{1} = \left(1 - \frac{1}{2}\right) - \left(-1 - \frac{1}{2}\right)$$

$$= \frac{1}{2} + \frac{3}{2} = 2$$

**19.** Area of the region bounded by the curve $y = \sqrt{49 - x^2}$ and the $x$-axis is

(*a*) $\frac{49}{2}\pi$ sq units (*b*) $98\pi$ sq units

(*c*) $49\pi$ sq units (*d*) $240\pi$ sq units

**Sol.** (*a*) as area is above the $x$-axis

$$\therefore \quad \text{area} = 2\int_0^7 \sqrt{49 - x^2}\,dx = 2\left[\frac{x}{2}\sqrt{49 - x^2} + \frac{49}{2}\sin^{-1}\frac{x}{7}\right]_0^7$$

$$= 2\left[\left(\frac{7}{2}\times 0 + \frac{49}{2}\sin^{-1}1\right) - (0)\right] = 49\frac{\pi}{2} \text{ sq units}$$

**20.** For the solution of differential equation $\frac{dy}{dx} + \frac{y}{x} = x^2$, the integrating factor is

(*a*) $\frac{y}{x}$ (*b*) $x$ (*c*) $\frac{1}{x}$ (*d*) $-x$

**Sol.** (*b*) $x$

## Section – B

**Section B consists of 20 questions of 1 mark each. Any 16 questions are to be attempted.**

**21.** Three balls are drawn from a bag containing 2 red and 5 black balls, if the random variable $X$ represents the number of red balls drawn, then $X$ can take values

(*a*) 0, 1, 2 (*b*) 0, 1, 2, 3 (*c*) 0 (*d*) 1, 2

**Sol.** (*a*), as there are 2 red balls, so maximum red balls can be 2.

**22.** The value of $p$ for which $\vec{a} = 3\hat{i} + 2\hat{j} + 9\hat{k}$ and $\vec{b} = \hat{i} + p\hat{j} + 3\hat{k}$ are parallel vectors is

(*a*) 3 (*b*) $\frac{3}{2}$ (*c*) $\frac{2}{3}$ (*d*) $\frac{1}{3}$

**Sol.** (*c*) as $\frac{3}{1} = \frac{2}{p} = \frac{9}{3} \Rightarrow p = \frac{2}{3}$

**23.** Distance between planes $\vec{r}\cdot(2\hat{i} + \hat{j} - 2\hat{k}) + 5 = 0$ and $\vec{r}\cdot(6\hat{i} + 3\hat{j} - 6\hat{k}) + 2 = 0$ is

(*a*) $\frac{9}{13}$ (*b*) $\frac{15}{4}$ (*c*) $\frac{13}{9}$ (*d*) $\frac{1}{13}$

**Sol.** (*c*) as distance $= \left|\frac{5 - \frac{2}{3}}{\sqrt{4 + 1 + 4}}\right|$

$$= \frac{13}{9} \text{ units}$$

**24.** $\int \frac{\sin^6 x}{\cos^8 x} dx$ is equal to

(*a*) $\frac{\sin^7 x}{\cos^9 x} + C$ (*b*) $\frac{1}{7}\tan^7 x + C$ (*c*) $\tan^6 x + C$ (*d*) $\sec^8 x + C$

**Sol.** (*b*) as $\int \tan^6 x \cdot \sec^2 x\, dx = \int t^6\, dt$

$$= \frac{t^7}{7} + C = \frac{\tan^7 x}{7} + C$$

| Let $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

**25.** If $\int \frac{2^x}{\sqrt{1-4^x}}\, dx = p.\sin^{-1}(2^x) + C$, then '$p$' is equal to

(*a*) $\log_e 2$ (*b*) $\frac{1}{2}\log_e 2$ (*c*) $\frac{1}{2}$ (*d*) $\frac{1}{\log_e 2}$

**Sol.** (*d*) let $2^x = t \Rightarrow 2^x \cdot \log_e 2\, dx = dt$

$$\therefore \quad \int \frac{2^x}{\sqrt{1-4^x}} dx = \frac{1}{\log_e 2} \cdot \int \frac{1}{\sqrt{1-t^2}} dt$$

$$= \frac{1}{\log_e 2} \cdot \sin^{-1}(2^x) + C$$

$$\Rightarrow \quad p = \frac{1}{\log_e 2}$$

**26.** If the area bounded by the curves $y^2 = 4ax$ and $y = mx$ is $\frac{a^2}{3}$, then the value of $m$ is

(*a*) 2 (*b*) –2 (*c*) $\frac{1}{2}$ (*d*) none of these

**Sol.** (*a*) $(mx)^2 = 4ax$

$$\Rightarrow \quad m^2x^2 = 4ax \quad \Rightarrow \quad x = 0,\ x = \frac{4a}{m^2}$$

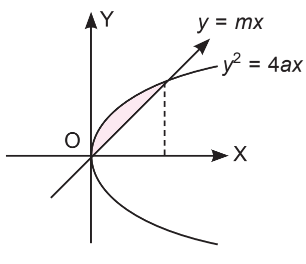
as the two curves intersect at 0, $\frac{4a}{m^2}$

$$\therefore \quad \text{Area} = \int_0^{\frac{4a}{m^2}} \left(\sqrt{4ax} - mx\right) dx$$

$$= \left[2\sqrt{a} \cdot \frac{2}{3}x^{3/2} - \frac{mx^2}{2}\right]_0^{\frac{4a}{m^2}}$$

$$= \frac{4\sqrt{a}}{3} \cdot \frac{4a}{m^2} \cdot \frac{2\sqrt{a}}{m} - \frac{m}{2} \cdot \frac{16a^2}{m^4}$$

$$= \frac{32a^2}{3m^3} - \frac{8a^2}{m^3} = \frac{8a^2}{3m^3}$$




Given $\frac{8a^2}{3m^3} = \frac{a^2}{3} \Rightarrow m^3 = 8 \Rightarrow m = 2.$

**27.** General solution of differential equation $\frac{dy}{dx} = x^5 + x^3 - \frac{2}{x}$ is

(*a*) $y = \frac{x^6}{6} + \frac{x^4}{4} - 2 \log |x|$ (*b*) $y = \frac{x^6}{6} + \frac{x^4}{4} - 2 \log |x| + 1$

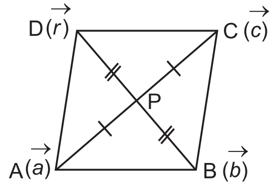
(*c*) $y = 5x^4 + 3x^2 + \frac{2}{x^2} + C$ (*d*) $y = \frac{x^6}{6} + \frac{x^4}{4} - 2 \log |x| + C$

**Sol.** (*d*)

**28.** If $\vec{a}, \vec{b}, \vec{c}$ be the position vectors of vertices *A*, *B*, *C* of a parallelogram *ABCD*, then the position vector of *D* is

(*a*) $\vec{a} + \vec{c} - \vec{b}$ (*b*) $\vec{a} - \vec{c} + \vec{b}$ (*c*) $\vec{a} - \vec{c} - \vec{b}$ (*d*) $\vec{c} - \vec{a} + \vec{b}$

**Sol.** (*a*) as in a parallelogram, diagonals bisect each other.




Using this concept, we get $\frac{\vec{a} + \vec{c}}{2} = \frac{\vec{b} + \vec{r}}{2}$

$\Rightarrow \quad \vec{r} = \vec{a} + \vec{c} - \vec{b}$

**29.** Equation of the line passing through the point (2, 1, 3) and perpendicular to the lines $\frac{x-1}{1} = \frac{y-2}{3} = \frac{z-3}{3}$ and $\frac{x}{-3} = \frac{y}{2} = \frac{z}{5}$ is

(*a*) $\frac{x-1}{2} = \frac{y-2}{-7} = \frac{z-3}{4}$ (*b*) $\frac{x}{-2} = \frac{y}{7} = \frac{z}{-4}$

(*c*) $\frac{x-2}{-2} = \frac{y-1}{7} = \frac{z-3}{-4}$ (*d*) none of these

**Sol.** (*d*) as line is $\frac{x-2}{a} = \frac{y-1}{b} = \frac{z-3}{c}$

and $a + 3b + 3c = 0$

$-3a + 2b + 5c = 0$

find *a*, *b*, *c*

**30.** *A* speaks truth in 70% cases and *B* speaks truth in 85% cases. The probability that they speak the same fact is

(*a*) 36% (*b*) 64% (*c*) 52% (*d*) 48%

**Sol.** (*b*) as $P(\text{same fact}) = P(AB \text{ or } \bar{A}\,\bar{B}) = \frac{70}{100} \times \frac{85}{100} + \frac{30}{100} \times \frac{15}{100}$

$= \frac{5950 + 450}{10000} = \frac{6400}{10000} = 64\%$

**31.** The position vectors of opposite vertices of a parallelogram are $2\vec{a} + 3\vec{b}$ and $\vec{a} - 2\vec{b}$. Then position vector of the point of intersection of diagonals is

(*a*) $3\vec{a} + \vec{b}$ (*b*) $\frac{\vec{a} + 5\vec{b}}{2}$ (*c*) $\frac{3\vec{a} + \vec{b}}{2}$ (*d*) none of these

**Sol.** (*c*)

**32.** The distance of the point (2, 3, 4) from the plane $\vec{r}\cdot(3\hat{i}-6\hat{j}+2\hat{k}) = -11$ is

(*a*) 1 unit (*b*) $\frac{1}{7}$ units (*c*) $\frac{11}{7}$ units (*d*) 2 units

**Sol.** (*a*) as distance of the point (2, 3, 4) from the plane $\vec{r}\cdot(3\hat{i}-6\hat{j}+2\hat{k})+11=0$ is

$$\left|\frac{(2\hat{i}+3\hat{j}+4\hat{k})\cdot(3\hat{i}-6\hat{j}+2\hat{k})+11}{\sqrt{9+36+4}}\right| = \left|\frac{6-18+8+11}{7}\right| = 1 \text{ unit}$$

**33.** Three persons *A*, *B* and *C*, fire a target in turn. Their probabilities of hitting the target are 0.2, 0.3 and 0.5 respectively, the probability that target hit is

(*a*) 0.993 (*b*) 0.94 (*c*) 0.72 (*d*) 0.90

**Sol.** (*c*) as

$$\begin{aligned} P(\text{target hit}) &= P(\text{at least one hits the target}) \\ &= 1 - P(\text{none hits}) \\ &= 1 - P(\bar{A}\ \bar{B}\ \bar{C}) \\ &= 1 - 0.8 \times 0.7 \times 0.5 = 0.72 \end{aligned}$$

**34.** If the direction cosines of a line are $\frac{k}{3}, \frac{k}{3}, \frac{k}{3}$, then value of *k* is

(*a*) $k > 0$ (*b*) $0 < k < 1$ (*c*) $k = \frac{1}{3}$ (*d*) $k = \pm\sqrt{3}$

**Sol.** (*d*) as $3 \times \frac{k^2}{9} = 1 \Rightarrow k = \pm\sqrt{3}$

**35.** The order of differential equation

$$y = \frac{dy}{dx} + \sqrt{1+\left(\frac{dy}{dx}\right)^3} \text{ is}$$

(*a*) 1 (*b*) 2 (*c*) 3 (*d*) none of these

**Sol.** (*a*)

**36.** If $|\vec{a}+\vec{b}| = |\vec{a}-\vec{b}|$, then angle between $\vec{a}$ and $\vec{b}$ is

(*a*) 0° (*b*) 90° (*c*) 180° (*d*) 60°

**Sol.** (*b*) as $|\vec{a}+\vec{b}|^2 = |\vec{a}-\vec{b}|^2 \Rightarrow (\vec{a}+\vec{b})^2 = (\vec{a}-\vec{b})^2 \Rightarrow \vec{a}\cdot\vec{b} = 0 \Rightarrow \theta = 90°$

**37.** The ratio in which the line segment joining the points (2, 4, 5) and (3, 5, – 4) is divided by *YZ*-plane is

(*a*) 5 : 4 internally (*b*) 4 : 5 externally (*c*) 2 : 3 externally (*d*) none of these

**Sol.** (*c*) if ratio is *k* : 1,

then $\frac{3k+2}{k+1} = 0 \Rightarrow k = -\frac{2}{3} \Rightarrow 2 : 3$ externally.

**38.** A card is picked at random from a pack of 52 playing cards. Given that the picked card is a queen, the probability of this card to be a card of spade is

(*a*) $\frac{1}{3}$ (*b*) $\frac{4}{13}$ (*c*) $\frac{1}{4}$ (*d*) $\frac{1}{2}$

**Sol.** (*c*) $P\left(\frac{\text{queen of spade}}{\text{queen}}\right) = \frac{\frac{1}{52}}{\frac{4}{52}} = \frac{1}{4}.$

**39.** $\int \frac{1}{\sqrt{4-x^2}} dx$ is equal to

(*a*) $\log|x + \sqrt{4-x^2}| + C$ (*b*) $\sin^{-1}\frac{2}{x} + C$

(*c*) $\sin^{-1}\frac{x}{2} + C$ (*d*) $\frac{1}{\log|x+\sqrt{4-x^2}|} + C$

**Sol.** (*c*) as $\int \frac{1}{\sqrt{4-x^2}} dx = \sin^{-1}\frac{x}{2} + C$

**40.** Solution of the differential equation $x\frac{dy}{dx} = y - x\tan\left(\frac{y}{x}\right)$ is

(*a*) $x \sin\frac{y}{x} = C$ (*b*) $\sin\frac{y}{x} = Cy$ (*c*) $\cos\frac{y}{x} = C$ (*d*) $x\cos\frac{y}{x} = C$

**Sol.** (*a*), as $\frac{dy}{dx} = \frac{y}{x} - \tan\left(\frac{y}{x}\right)$, homogeneous equation

Let $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$v + x\frac{dv}{dx} = v - \tan v \Rightarrow \int \frac{1}{\tan v} dv = -\int \frac{1}{x} dx$

$\Rightarrow \int \cot v \, dv = -\int \frac{1}{x} dx \Rightarrow \log|\sin v| = -\log|x| + \log C$

$\Rightarrow \log|\sin v| = \log\left|\frac{C}{x}\right| \Rightarrow x\sin\frac{y}{x} = C$ is the required solution.

## Section – C

**Section C consists of 10 questions. Attempt any 8 questions.**

**Questions 46-50 are based on a case-study.**

**41.** A pair of dice is thrown and it is known that the second dice always exhibits an odd number. Then the probability that the sum obtained on two dice is 7, is

(*a*) $\frac{1}{6}$ (*b*) $\frac{5}{6}$ (*c*) $\frac{1}{2}$ (*d*) none of these

**Sol.** (*a*) as sum is 7 → 16, 25, 34, 43, 54, 61

second dice odd no →11, 21, 31, 41, 51, 61

13, 23, 33, 43, 53, 63

15, 25, 35, 45, 55, 65

Probability $= \frac{3}{18} = \frac{1}{6}$

**42.** Distance between parallel planes $2x - y + 3z - 4 = 0$ and $6x - 3y + 9z + 13 = 0$ is

(*a*) 17 (*b*) $\frac{25}{3}$ (*c*) $\frac{25}{3\sqrt{14}}$ (*d*) $\frac{25}{\sqrt{14}}$

**Sol.** (*c*)

**43.** The probability distribution of the discrete variable $X$ is given as:

| $X$ | 2 | 3 | 4 | 5 |
|---|---|---|---|---|
| $P(X)$ | $\frac{5}{k}$ | $\frac{7}{k}$ | $\frac{9}{k}$ | $\frac{11}{k}$ |

The value of $k$ is

(*a*) 8 (*b*) 16 (*c*) 32 (*d*) 48

**Sol.** (*c*), as $\Sigma P(X) = 1 \Rightarrow \frac{5}{k} + \frac{7}{k} + \frac{9}{k} + \frac{11}{k} = 1 \Rightarrow k = 32$

**44.** The distance of the point (3, – 5, 12) from the $x$-axis is

(*a*) 13 units (*b*) 10 units (*c*) 9 units (*d*) 144 units

**Sol.** (*a*), Distance of the point (3, – 5, 12) from the $x$-axis $= \sqrt{(-5)^2 + (12)^2} = \sqrt{25 + 144}$

$= \sqrt{169} = 13$ units

**45.** The value of $\lambda$ for which vectors $2\hat{i} + \hat{j} + 3\hat{k}$ and $\hat{i} - \lambda\hat{j} + 4\hat{k}$ are orthogonal is

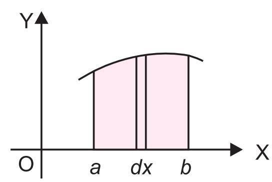
(*a*) 12 (*b*) –12 (*c*) 14 (*d*) –14

**Sol.** (*c*) as $(2\hat{i} + \hat{j} + 3\hat{k}) \cdot (\hat{i} - \lambda\hat{j} + 4\hat{k}) = 0$ $\Rightarrow 2 - \lambda + 12 = 0$ $\Rightarrow \lambda = 14$

## Case-Based Questions

Area bounded by the curve $y = f(x)$, the $x$-axis and between the ordinates at $x = a$ and $x = b$ is given by

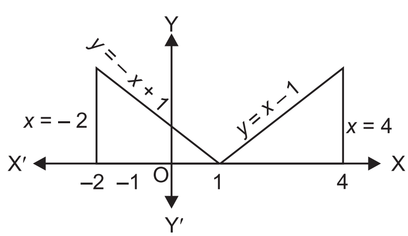
Area $= \int_a^b y\,dx = \int_a^b f(x)\,dx$



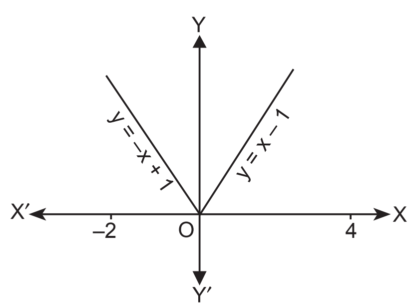

Based on the above information, answer the following:

**46.** The graph of $y = |x - 1|$ is

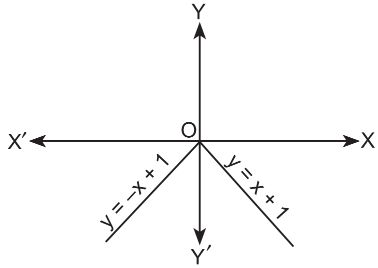
(*a*)



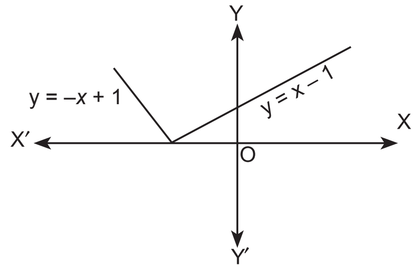

(*b*)



(*c*)

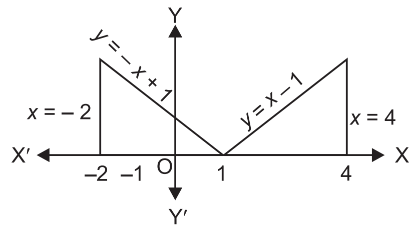



(*d*)




**Sol.** (*a*)

$$y = |x - 1|$$
$$y = -x + 1 \quad \text{or} \quad y = x - 1$$




**47.** The value of $\int |x-1|\, dx$ is

(*a*) $\dfrac{|x+1|(x-1)}{2}+C$ (*b*) $\dfrac{|x-1|(x-1)}{2}+C$ (*c*) $\dfrac{(x-1)^2}{2}+C$ (*d*) $|x-1|+C$

**Sol.** (*b*) Let $|x-1| = \begin{cases} (x-1) & x \geq 1 \\ -(x-1) & x < 1 \end{cases}$

$$\int |x-1|\, dx = \int (x-1)\, dx \text{ or } -\int (x-1)\, dx = \frac{(x-1)^2}{2}+C \text{ or } \frac{-(x-1)^2}{2}+C$$
$$= \frac{(x+1)(x-1)}{2} \text{ or } \frac{-(x-1)(x-1)}{2} = \frac{|x-1|\,(x-1)}{2}.$$

**48.** The value of $\int_{-2}^{4} |x-1|\, dx$ is

(*a*) 4 sq units (*b*) 3 sq units (*c*) 9 sq units (*d*) 10 sq units

**Sol.** (*c*), $\int_{-2}^{4} |x-1|\, dx = \int_{-2}^{1} -(x-1)\, dx + \int_{1}^{4} (x-1)\, dx = \left[\frac{-x^2}{2}+x\right]_{-2}^{1} + \left[\frac{x^2}{2}-x\right]_{1}^{4}$

$$= \left(-\frac{1}{2}+1\right) - \left(-\frac{4}{2}-2\right) + \left(\frac{16}{2}-4\right) - \left(\frac{1}{2}-1\right) = 9 \text{ sq units.}$$

**49.** The coordinates of point at which the graph of $y = |x - 1|$ crosses the $y$-axis is

(*a*) (0, 2) (*b*) (0, 1) (*c*) (0, –1) (*d*) (0, 3)

**Sol.** (*b*) We have $y = \begin{cases} x-1, & x \geq 1 \\ -x+1, & x < 1 \end{cases}$

When $x = 0$

$\Rightarrow$ $y = -0 + 1 = 1$ point is (0, 1)

**50.** The value of the definite integral on the graph represents

(*a*) volume bounded by the curve (*b*) density bounded by the curve

(*c*) area bounded by the curve (*d*) length of the curve

**Sol.** (*c*), The value of the integral represents the area bounded by the curve, the $x$-axis and between ordinates at $x = a$, $x = b$.